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ORIGINAL ARTICLE

Long-Run Expectations, Learning and the US Housing Market

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Abstract
In the US housing market, the price-to-rent ratio is volatile and autocorrelated. Returns on housing are positively autocorrelated. The price-to-rent ratio is negatively correlated with future returns and rent growth. Housing returns exhibit time-varying volatility. A benchmark asset pricing model is inconsistent with these facts. A model where prices adjust slowly to their fundamental value and where the agent does not know whether housing fundamentals are trend or difference stationary so has changing beliefs over time, increases the volatility of prices and the autocorrelation of returns. The price-to-rent ratio negatively forecasts returns and rent growth. This model generates time-varying volatility.

Keywords Learning · Expectations · Housing demand · Asset pricing

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Introduction

Given the recent boom and bust in housing markets, there is renewed interest in understanding the determinates of US house prices. In this paper I begin by noting that housing market data present several puzzles that are present in asset

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market data more broadly, especially equities. First, there is evidence of excess volatility in house prices. The standard deviation of the price-to-rent ratio is 15% and the standard deviation of housing returns is 6%, while the standard deviation of the underlying housing rents is only 2.3%. Second, housing returns are significantly positively autocorrelated. Finally, the price-to-rent ratio is negatively correlated with future returns and rent growth and housing returns show evidence of time-varying volatility. While housing as an asset is in many ways different than equities, the similarities between these empirical facts and facts in the aggregate US equity market motivate me to examine the equilibrium housing price in a general equilibrium asset pricing framework.[1,2]

I begin with a standard consumption-based asset pricing model and use a log-linear approximation to the Euler equation as in Campbell (1993) and Restoy and Weil (2011) to solve for the equilibrium house price. The equilibrium house price then depends on future expectations of fundamentals (housing preferences) and consumption. I show that this model is unable to explain the facts outlined in the previous paragraph. I then examine the ability of two assumptions to bring the rational expectations model closer to the data. First, I allow for prices to adjust slowly to their fundamental value. Second, I assume the agent does not know the true model for housing preference shocks. Specifically, they are unsure whether preference shocks are permanent or temporary. They use a Bayesian learning model as in Cogley and Sargent (2005) to learn whether the preference process is trend stationary or difference stationary. Their beliefs change over time depending on how well each model fits the data. While the true process is (trend) stationary, the agent does not know this. He puts excessive weight on the difference stationary process, overreacting to temporary changes in market preferences.

Two features of the housing preference processes make this learning significant. The first is the well-known fact that unit root and near unit root processes are very difficult to tell apart in small time series sample (Cochrane 1988; Stock 1991). As a result, the agent will almost always put some weight on the difference stationary process even if the true process is trend stationary. Additionally, after a random sequence of shocks which moves housing preferences away from their long-run trend the agent will put additional weight on the difference stationary model. Second, analogous to the analysis of the permanent income hypothesis (Deaton 1992) if the individual believes the true process is a unit root process, then shocks are permanent. As a result, they will react strongly to news about fundamentals and return volatility will increase.

The sticky price assumption and the learning mechanism allow the model to better match the data, though the success is somewhat limited. Learning amplifies volatility over the rational expectations benchmark. I find that the learning model generates four times the amount of volatility in both housing returns and

1 For an analysis of similar facts for the S&P 500 index, see Fuster et al. (2012) or Tortorice (2018).

2 Some of the important differences between housing markets and equity markets include larger transactions costs for buying and selling houses, the use of debt to purchase housing and the fact that homeowners usually own exactly one house at a time.

the HP-filtered house price as compared to the rational expectations model. On the other hand, the learning model does not amplify the standard deviation of the price-to-rent ratio over the rational expectations model, and the price-to-rent ratio is five times as volatile in the data than in the model. Sticky prices allow the model to match the autocorrelation of housing returns, though learning dampens this autocorrelation.

The learning model improves more substantially over the rational expectations model when we examine the role the price-to-rent ratio has in predicting future returns and rent growth. Consistent with the data, learning creates a negative correlation between the price-to-rent ratio and future returns and rent growth. The rational expectations model predicts essentially a zero correlation between the price-to-rent ratio and future returns and a positive correlation between the price-to-rent ratio and future rent growth. Additionally, the learning model generates time-varying volatility as in the data. The learning model generates excessive kurtosis of returns. Data from the learning model are consistent with the positive autocorrelation of squared residuals from an AR(1) regression of returns and the estimation of GARCH effects in US data. The rational expectations variant of the model predicts these values should all be zero.

The mechanism by which learning brings the model closer to the data is straightforward. First, since the house price is the present discounted value of future fundamentals (housing preferences), house price volatility is directly affected by the nature of the process for preferences. If preferences are trend stationary, then shocks to preferences have only a temporary effect on fundamentals and they quickly return to trend. As a result, the present discounted value of fundamentals does not change very much. On the other hand, if the true process is non-stationary, shocks have permanent effects, and the present discounted value of fundamentals responds more to the shocks. The result is that house prices are more volatile under the non-stationary process than the trend stationary process. Allowing the agent to entertain the possibility that fundamentals are non-stationary results in him reacting more strongly to shocks to fundamentals than in the rational expectations world where the process for fundamentals is trend stationary and the agent knows this. Consequently, learning is able to amplify volatility. This same mechanism explains why the model exhibits time-varying volatility. Since the agent's probability weight on the non-stationary model varies over time, the magnitude of his reaction to shocks will vary over time as well, creating time-varying volatility.

The learning mechanism is also able to generate return predictability with the price-to-rent ratio. This result occurs because agents are incorrectly forecasting future fundamentals. After a series of random but positive shocks to fundamentals, housing preferences drift away from trend. As a result, the agent puts more weight on the non-stationary model versus the stationary model. These beliefs lead him to overestimate the persistence of today's positive fundamentals. As a result, the price rises and the price-to-rent ratio is high. But then fundamentals return to trend, leading the agent to revise down his belief in the non-stationary model, lowering the house price and leading to lower returns.

As one can tell, the main goal of this paper is understanding changes in house prices. While the goal of better understanding house price volatility could be interesting in and of itself, there are also important practical reasons to study the question. As first emphasized by Shiller and Weiss (1999), the currently 20 trillion-dollar investment in residential housing is one that is very difficult to hedge. Few if any derivatives exist to mitigate the risk of house prices fluctuations. As a result, housing for many consumers remains a highly illiquid, geographically undiversified, leveraged investment. Understanding the drivers of house price volatility then helps us to understand the nature of risks that many households face on their largest source of wealth. Additionally, the financial system, especially the banking sector, is often exposed to house price risk through mortgage lending. A better understanding of house price volatility contributes to our understanding of the fragility of the financial system.

Many papers have tried to explain the recent US housing boom and bust in rational expectations models using various institutional features and frictions in the housing market, like changes in down-payment requirements (Chu 2014; Chambers et al. 2009; Iacoviello and Pavan 2013; Corbae and Quintin 2015; Garriga and Schlagenhauf 2009; Titman et al. 2014; Chatterjee and Eyigungor 2009; Favilukis et al. 2017). While these models have considerable success explaining the volatility of house prices and returns, they generate return predictability in housing returns in an unsatisfactory way. To the extent there is return predictability, agents are aware of this predictability and therefore expectations of future returns are low when the price-to-rent ratio is high. This result is in contrast to expected returns in my model and data on survey expectations discussed below.[3]

One of the first papers to examine data on house price expectations is Case and Shiller (2003). They find that home buyers have unrealistic expectations concerning future house price increases, predicting double digit increases annually over the next 10 years. Households also are unlikely to view housing as a risky investment. Case et al. (2012) replicate these results and argue that long-run expectations (10-year) and house price expectations are the primary driver of the house price boom. Piazzesi and Schneider (2009) argue for the presence of momentum traders in the housing market, agents who are always optimistic about price changes. Foote et al. (2012) present evidence that even industry financial analysts were bullish about house prices even at the 2006 house price peak.

Based on this empirical evidence, many authors have examined the implication of relaxing rational expectations for house price dynamics. In fact, given the shortcomings of rational expectations model to match the volatility of house prices, Glaeser and Gyourko (2006) and Glaeser et al. (2008) argue that deviations from rational expectations and models of learning may be fruitful avenues of research as do Piazzesi and Schneider (2016). Glaeser and Nathanson (2017) study a model where agents neglect to consider the forecasts of other agents when forecasting future prices. Burnside et al. (2016) consider a model where, as in the current paper,

[3] This critique of pure rational expectations models extends to the literature incorporating housing into macroeconomic models, for example, Benhabib et al. (1991), Greenwood and Hercowitz (1991), Davis and Heathcote (2005) Iacoviello (2005), Iacoviello and Neri (2010) and Miao et al. (2014). However, these papers do find it hard to generate house price volatility motivating the work in this paper.

learning about long-run fundamentals is essential. However, in their paper learning comes from social dynamics as opposed to observation of fundamentals. Bolt et al. (2014) generated boom and busts in house prices through an heterogeneous agent model where the agents endogenously switch between different price forecast rules. Adam et al. (2012) consider a model where agents learn about house price growth in an open economy model. Caines (2015) incorporates the Adam et al. model into a model where expectations affect both the demand and supply sides of housing. He finds that a substantial fall in the mortgage rate can replicate key fact of the recent US housing boom. Gelain and Lansing (2014) consider learning in an asset pricing-based model of housing where agents learn about rent growth using a misspecified model and extrapolative expectations. Similarly, Granziera and Kozicki (2015) explore the role of bubbles and extrapolation for explaining the volatility of house prices. All these models increase volatility of the price-to-rent ratio, and the Gelain and Lansing paper also generates predictability in house price returns. However, this paper differs from these in important ways. First, the paper seeks to endogenously explain both the predictability and the time-varying volatility in housing returns as well as amplifying volatility.[4] Secondly, I present a novel model of learning where agents are unsure about the true process for fundamentals and change their beliefs based on how accurately each model captures the data. As such, the model here provides a theoretical justification for extrapolative beliefs as opposed to simply assuming extrapolation exogenously. Finally, agents make significant mistakes about their long-run expectations (as opposed to their short-run expectations) consistent with the results in Case et al. (2012, 2014).[5]

Finally, while it might seem simplistic to model housing in a purely asset pricing framework as opposed to a supply and demand framework, there is a large literature that takes the asset pricing approach.[6] For example, Piazzesi et al. (2007) model housing jointly as an asset and a consumption choice in an otherwise standard consumption-based asset pricing model. They show that housing increases the risk premium and predicts excess returns in equity markets. Lustig and Van Nieuwerburgh (2005) reach a similar conclusion in a model where housing is an important source of collateral. Flavin and Nakagawa (2008) explore how the illiquidity of housing influences the stochastic discount factor in a consumption-based asset pricing model. Ayuso and Restoy (2006) apply the asset pricing framework of Restoy and Weil (2011) and show that a large part of the fluctuations in Spanish house prices cannot be explained with observed fundamentals. The present paper uses the asset pricing framework of Restoy and Weil (2011) to explain house prices, but differs from the above papers by focusing on the volatility and predictability of housing returns and considering a learning-based model of expectation formation.

4 In Gelain and Lansing (2014) for example, fundamentals exhibit exogenous time-varying volatility.

5 Nguyen (2014) uses a similar learning model to explain serially correlated house price forecast errors and house price volatility in a model in which housing is allocated by a central planner who does not know the true process for housing preference shocks.

6 For example of the supply and demand approach, see Wheaton (1990), Krainer (2001), Glaeser and Gyourko (2006) and Head et al. (2012).

The rest of the paper proceeds as follows. Section two discusses the data and the key empirical facts. Section three outlines the model and section four explains its calibration. Section five gives the main model results, and section six demonstrates the robustness of the results to alternative parameter specifications. Section seven concludes.

Data

Data come from Davis et al. (2008).[7] Data begin in 1960:Q1 and end in 2013:Q1. Data on rents and house prices are obtained from the Decennial Census of Housing from 1960 to 2000. Data on rents are interpolated between census dates using the Bureau of Labor Statistics (BLS) index for the rent of primary residences. Data on house prices are interpolated using the Freddie Mac (CMHPI) series repeat-sales house prices index after 1970 and the median price of new homes sold index before 1970. The Macromarkets LLC national house price index, formally known as the Case–Shiller–Weiss index, is used after 2000 to construct house prices. Prices are deflated using the CPI.

These data have important strengths that are key to accomplishing the goals of this paper. They report, on a quarterly basis, both a dollar value for average rents and house prices. This time series is also quite long, over 50 years. The decennial censuses, which are used to benchmark the data, is high quality being a very large sample, 1% of the US population. The data use repeat-sales indices to extrapolate prices when available and the data are publicly available. The data are not without issues though. The 1970 census does not include values for owner-occupied multi-family dwellings and does not include data on utilities to subtract out of gross rents. Finally, house prices are top-coded in each of the censuses. The authors deal with these data limitations by imputing the data from the other censuses and using data from the Survey of Consumer Finances.

Moments for the data are given in Table 1. The full sample expected return on housing, given by $E\frac{q_t}{q_{t-1}-d_{t-1}}$ where q_t is the house price at time t and d_{t-1} is the rent at time $t-1$, is equal to 6.4% on an annual basis. The standard deviation of the annual return is 6%. Rent growth averages 1% per year with a standard deviation of 2.3%. These data indicate the presence of an excess volatility puzzle with returns being almost three time as volatile as the underlying fundamental rents. The standard deviation of the log price-to-rent ratio is 15%, and the standard deviation of the log HP-filtered housing price is 3.7%.[8]

Examining autocorrelations of the data at one to four quarterly lags, we see that the price-to-rent ratio is highly persistent with all autocorrelation coefficients above 0.95. The autocorrelation of returns declines from 0.84 at one lag to 0.51 at four

[7] Data are available at: http://www.lincolninst.edu/subcenters/land-values/rent-price-ratio.asp.

[8] The price-to-rent ratio is calculated as q_t/d_t. There is no need to take a past average of rents as rents (as opposed to equity dividends) are quite smooth in the data.

Table 1 Data moments

Means and standard deviations		Skewness and kurtosis	
$E(r_t)$	0.064	skew(P_t/R_t)	2.04
$E(\Delta \ln(\text{rent}_t))$	0.009	skew(r_t)	− 1.7
$\sigma(P_t/R_t)$	0.15	kurtosis(P_t/R_t)	7.1
$\sigma(P_t^{\text{HP}})$	0.037	kurtosis(r_t)	7.4
$\sigma(r_t)$	0.06	Predictability	
$\sigma(\Delta \ln(\text{rent}_t))$	0.023	$\rho(P/R_t, r_{t+1} + \cdots r_{t+16})$	− 0.74
		$\rho(r_t, r_{t+1} + \cdots r_{t+16})$	0.32
		$\rho(P/R_t, \ln(\text{rent}_{t+16}) - \ln(\text{rent}_t))$	− 0.44
Autocorrelations		Squared autocorrelations	
$\rho(P_t/R_t)$	0.99	r_t^2	0.7
	0.98		0.56
	0.95		0.45
	0.92		0.27
$\rho(r_t)$	0.84	$(\epsilon_t^r)^2$	0.14
	0.73		0.39
	0.66		0.1
	0.51		0.32
		GARCH model	
		Υ_1 (GARCH)	0.73
			(0.06)
		a_1 (ARCH)	0.27
			(0.07)
		p value Engle test	0.04

This table provides the key moments on house prices, rents and price-to-rent ratio. For the GARCH model coefficient standard errors are given in parentheses

lags. The existence of positive momentum in the housing market has previously been documented by Case and Shiller (1989) among others.

Additionally, there is evidence of return predictability in the housing data. The price-to-rent ratio is negatively correlated with the cumulative return over the next 4 years $r_{t+1} + \cdots + r_{t+16}$ with a correlation coefficient of -0.74. However, the current period return is positively correlated with the same cumulative return with a coefficient of 0.19. Finally, the price-to-rent ratio is negatively correlated with future rent growth, $\ln d_{t+16} - \ln d_t$ with a coefficient of -0.44. This long-term mean reversion in housing prices is also noted by Glaeser et al. (2014).[9]

As a further examination of return predictability in the data, I estimate the following Campbell–Shiller (1988) style regressions:

$$r_{t+1} + \cdots + r_{t+16} = \alpha + \beta^{\text{return}} \ln(\text{price-to-rent}_t) + \epsilon_t$$
$$\ln d_{t+16} - \ln d_t = \alpha + \beta^{\text{rent}} \ln(\text{price-to-rent}_t) + \epsilon_t$$

[9] Results are qualitatively similar for a variety of horizon windows from 3 years onward.

Table 2 Campbell–Shiller regressions

Future return regression ($r_{t+1} + \cdots + r_{t+16}$)	
β	R^2
− 0.8	0.5
(0.06)	
Future rent growth regression ($\ln \xi_t + \cdots \ln \xi_{t+16}$)	
β	R^2
− 0.12	0.2
(0.02)	

This table presents results from running Campbell–Shiller regressions on the housing data. The price-to-rent ratio is used to predict returns and rent growth over the next 4 years. Standard errors are in parentheses

Results for the regressions are presented in Table 2. We find that $\beta^{\text{return}} = -0.8$. This coefficient implies that a 10% increase in the price-to-rent ratio predicts cumulative returns will be −8% lower over the next 4 years or about −2% per year. The R-square of this regression is 0.5, suggesting cumulative returns over the next 4 years are explained fairly well using the price-to-rent ratio. On the other hand, $\beta^{rent} = -0.12$ and the R-square of the regression is only 0.2. This suggests that rent growth is less predictable than future returns and importantly high price-to-rent ratios do not seem to forecast periods of high demand for housing. Quite the opposite, if anything, they predict we are entering a period of low demand for housing.

There is also substantial evidence of time-varying volatility in the data. I report skewness $E\frac{(x-\mu)^3}{\sigma^3}$ and kurtosis $E\frac{(x-\mu)^4}{\sigma^4}$ of the price-to-rent ratio. Housing returns are left-skewed with a skewness of -2, while the price-to-rent ratio is right-skewed with a skewness of 2.04. Both series also demonstrate high levels of kurtosis of around 7. If the series were normally distributed, they would exhibit a kurtosis of 3. The high level of kurtosis is evidence of the existence of fat-tails in the distribution, i.e., increased frequency of extreme values relative to a normal distribution.

As further evidence of time-varying volatility, I examine the autocorrelation of squared returns and the autocorrelations of squared residuals from an AR(1) return regression, i.e., ϵ_t^2 from $r_t = \alpha + \rho r_{t-1} + \epsilon_t$. If large returns and residuals are more likely followed by large returns and residuals, as would be the case with time-varying volatility, we would expect to see positive autocorrelation of squared returns and residuals. That is indeed what we see. The autocorrelation of squared returns ranges from 0.7 at one lag to 0.27 and four lags. Similarly, the autocorrelation of squared residuals ranges from 0.1 to 0.39.[10]

For additional evidence of time-varying volatility in the aggregate US housing data, I estimate a GARCH(1,1) model on the quarterly return series. The GARCH(1,1) model is:

$$\sigma_t^2 = \kappa + \gamma_1 \sigma_{t-1}^2 + a_1 \epsilon_{t-1}^2.$$

[10] Under the null hypothesis of zero autocorrelation, standard errors are calculated as $\frac{1}{\sqrt{T}}$= 0.07, making these estimates statistically significant. See Hamilton (1994, p. 111).

In this model the variance of ε_t in the AR(1) regression $r_t = \alpha + \rho r_{t-1} + \varepsilon_t$ is varying over time. Positive γ_1 and a_1 are evidence of time-varying volatility with data that will exhibit periods of particularly high volatility. For the quarterly return data, I estimate $\gamma_1 = 0.73$ and $a_1 = 0.27$. Both estimates are highly statistically significant. Furthermore, the Engle test (1982) rejects the null of no GARCH effects at the 95% confidence level.

Model

Housing Price

A representative consumer can consume or buy units of housing h_t at a price q_t. The household is subject to stochastic shocks to their preference for housing ξ_t. The household's problem then is to choose

$$\max_{c_t, h_t} E_o \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\gamma}}{1-\gamma} + \xi_t \ln h_t \right]$$

subject to the constraint:

$$c_t + q_t h_t = q_t h_{t-1} + y_t. \tag{1}$$

Here y_t is income at time t, and c_t is consumption at time t. The first-order conditions for the consumer's optimal choice are:

$$c_t : c_t^{-\gamma} - \lambda_t = 0 \tag{2}$$

$$h_t : \frac{\xi_t}{h_t} - \lambda_t q_t + \beta E_t \left[\lambda_{t+1} q_{t+1} \right] = 0 \tag{3}$$

where λ_t is the Lagrange multiplier on the budget constraint.

Combining these two equations, one gets

$$q_t = \frac{\xi_t}{h_t \lambda_t} + \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} q_{t+1} \right]$$

I will assume that housing is in fixed supply so that $h_t = 1$ for all t. While I will refer to h_t as housing, it is more accurate to define h_t as land, since land is naturally in fixed supply while housing can be produced. Then, from the first-order condition we have:

$$q_t = \frac{\xi_t}{\lambda_t} + \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} q_{t+1} \right].$$

Letting $d_t = \frac{\xi_t}{\lambda_t}$, we can write

$$1 = \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{q_{t+1}}{q_t - d_t} \right]$$

note that we interpret d_t as housing rents because it is the utility dividend paid to the holder of the housing asset.

This is a highly nonlinear condition. But, following Campbell (1993) and Restoy and Weil (2011), it can be linearized in two steps. First assuming that housing returns and consumption are conditionally homoscedastic and jointly log-normally distributed, we obtain:

$$\ln(q_t - d_t) = \ln\beta + E_t\left[-\gamma\Delta\ln c_{t+1} + \ln q_{t+1}\right] + \frac{1}{2}\left[\gamma^2\sigma_c^2 + \sigma_q^2 - 2\gamma\sigma_{c,q}\right] \tag{4}$$

where $\sigma_c^2 = Var_t\Delta\ln c_{t+1}, \sigma_q^2 = Var_t\ln q_{t+1}$ and $\sigma_{c,q} = Cov_t(\Delta\ln c_{t+1}, \ln q_{t+1})$.

Then I linearize $\ln(q_t - d_t)$ about the steady rent-to-price ratio and obtain:

$$\ln(q_t - d_t) \approx k + (1-\delta)\ln q_t + \delta\ln d_t \tag{5}$$

where $\bar{x} = E\ln(\frac{d_t}{q_t})$, $k = \ln(1-\exp(\bar{x})) + \frac{\exp(\bar{x})}{1-\exp(\bar{x})}\bar{x}$ and $\delta = \frac{-\exp(\bar{x})}{1-\exp(\bar{x})}$.

Substituting (5) into (4) and letting $\sigma = \frac{1}{2}\left[\gamma^2\sigma_c^2 + \sigma_q^2 - 2\gamma\sigma_{c,q}\right]$,we have:

$$\ln q_t \approx \frac{1}{1-\delta}\left[\sigma + \ln\beta - k - \delta\ln d_t + E_t(-\gamma\Delta\ln c_{t+1} + \ln q_{t+1})\right].$$

Iterating forward

$$\ln q_t \approx \frac{k - \ln\beta - \sigma}{\delta} - \frac{\delta}{1-\delta}E_t\sum_{s=0}^{\infty}\frac{1}{(1-\delta)^s}\ln d_{t+s} - \frac{\gamma}{1-\delta}E_t\sum_{s=0}^{\infty}\frac{1}{(1-\delta)^s}\Delta\ln c_{t+s+1}$$

Since $d_t = \frac{\xi_t}{\lambda_t}$, we obtain $\ln d_t = \ln\xi_t - \ln\lambda_t = \ln\xi_t + \gamma\ln c_t$. Therefore, we can solve for the following closed-form solution for the log house price:

$$\begin{aligned}\ln q_t \approx{}& \frac{k - \ln\beta - \sigma}{\delta} - \frac{\delta}{1-\delta}E_t\sum_{s=0}^{\infty}\left[\frac{1}{(1-\delta)^s}\left(\ln\xi_{t+s} + \gamma\ln c_{t+s}\right)\right] \\ &- \frac{\gamma}{1-\delta}E_t\sum_{s=0}^{\infty}\frac{1}{(1-\delta)^s}\Delta\ln c_{t+s+1}\end{aligned} \tag{6}$$

The online supplementary material contains analytical expressions for the sums and the conditional variances as a function of the underlying processes for consumption growth and housing preferences listed below.[11]

Iacoviello (2010) argues that institutional rigidities lead to sluggish adjustment in the housing market. For example, listing agents often used “comps” or comparable sales when setting prices. These are listings which are similar to the property for sale and have sold recently. Likewise, banks often will not grant a mortgage on a property that does not appraise for the sale price. I model these institutional features

[11] In general, these sums are finite given the calibration as $(1-\delta) > 1$ and the presence of the *log* of the variable in the numerator. To see this let $x_{t+s} = (1+g_x)^s x_t$. Then $\ln x_{t+s} = sln(1+g_x)$and $\sum_{s=0}^{\infty}\frac{s}{(1-\delta)^s} = \frac{1-\delta}{\delta^2}$for $(1-\delta) > 1$.

in a reduced form way, assuming rigidity in the price such that the market price q^* is given by:

$$\ln q^*_t = \lambda \ln q_t + (1-\lambda)\ln q^*_{t-1} \tag{7}$$

where λ is a measure of the degree of institutional price rigidity in the market. In this formulation, the market price is assumed to slowly adjust to the fundamental price. The smaller is λ, the more sluggishly prices adjust in the housing market.[12]

To close the model, it is necessary to specify the exogenous processes for income, consumption and housing preference. First, note that given the budget constraint ($c_t + q_t h_t = q_t h_{t-1} + y_t$) and the assumption that $h_t = 1$ for all t, we have that $c_t = y_t$ for all t. Therefore, one can specify either the income or consumption process. Here, I specify the consumption process. In order to focus on the role of future expectations of housing fundamentals for driving house prices, I assume that consumption growth is i.i.d. though it can be correlated with the housing preference process as described in the calibration section. Again, to focus on learning about future housing fundamentals, I allow the consumer to know the exact process for consumption.

In contrast, there is uncertainty about the nature of the housing preference process. The actual law of motion (ALM) for the housing preference process will be the trend stationary process given by Eq. (8). However, the consumer does not know the true form of the preference process ξ_t. Specifically, he is uncertain whether the preference process is trend stationary or not. His perceived law of motion (PLM) for the housing preference process is:

$$\xi_t = \rho^s_0 + \beta t + \rho^s_1 \xi_{t-1} + \cdots + \rho^s_p \xi_{t-p} + \varepsilon^s_t \tag{8}$$

with probability p^s_t and that

$$\Delta\xi_t = \alpha + \rho^{ns}_1 \Delta\xi_{t-1} + \cdots + \rho^{ns}_p \Delta\xi_{t-p} + \varepsilon^{ns}_t \tag{9}$$

with probability $p^{ns}_t = 1 - p^s_{t.}$ These beliefs will vary over time given how well each model fits the data as described in the following subsection.[13]

In this paper, consistent with much of the literature on macroeconomic models of house prices, shocks to housing preferences are a key driver of fluctuations. Iacoviello and Neri (2010), Adam et al. (2012) and Liu et al. (2013) among others all rely on preference shocks to generate volatility in house prices.[14] While the literature is vague about what the housing preference process represents, it is intended to be a reduced form representation of the various shocks which can affect house

[12] For a complete theory of nominal rigidities with strategic complementarities leading to sluggish pricing in housing markets, see Guren (2014).

[13] While the main model is quite simple, the online supplementary material demonstrates that Eq. 6 can be generated in more complicated models. For example, ones with variable labor supply, quadratic adjustment costs to housing, and depreciation of the housing stock. Therefore, I have not found that these modifications alter the model fit. This is not to say that the model considers all complications. Chakraborty (2016), for instance, considers land prices in rational expectations, DSGE model with credit constraints and permanent shocks to TFP and land taxes and finds additional amplification.

[14] Other examples of papers with exogenous fluctuations in housing preferences include Chambers et al. (2009), Miao et al. (2014), Guerrieri and Iacoviello (2013), Sterk (2015), Lambertini et al. (2013) and Filipa and Wieladek (2015).

prices. It could represent interest rates, credit availability as well as a strict preference shock to the demand for housing. It can also represent macroeconomic conditions and housing supply conditions which affect house prices. Modeling these factors in a reduced form way allows the model to better focus on the role learning and expectations can have in driving house prices.

However, it is worth noting that the emphasis on housing preference shocks receives strong support in the literature. In a general equilibrium, open economy model, Filipa and Wieladek (2015) find that a housing preference shock is an important determinate of house prices. Iacoviello and Neri (2010) also find a large fraction of the variation in house prices can be attributed to preference shocks, even controlling for a wide variety of fundamentals like income, interest rates, availability of subprime mortgages and mortgage fees. They then show that their estimated preference shocks are correlated with many of the factors we believe drive housing markets. These factors include fees to purchase a home, inflation, fraction of subprime mortgages and demographic variables. Time-varying importance of housing for utility is also supported by Iacoviello (2010) who emphasizes that many press articles explain changes in house prices with changes in the nature of what housing consumers are looking for, e.g., looking for larger homes or viewing homes as an investment vehicle. In addition to this empirical evidence, Iacoviello and Neri (2010) also provide a theoretical explanation for time-varying housing preference. The first is a model where the housing stock delivers time-varying housing services, and the second is a model where the resources, in terms of other goods, required to purchase a house is varying over time.

An important question here is the order of integration of the preference process: is it trend or difference stationary? While most papers assume a stationary preference process, this is not always supported by the literature. Adam et al. (2012) use a unit root process for housing preferences. They then show that after estimating the preference process from rental data, across six different countries, they are unable to reject the possibility of a unit root in any of the countries. It is also natural to assume some type of trend in preference for housing, either stochastic or deterministic, to match the growth of housing fundamentals (rents) in the data.

Finally, given the broad interpretation of the preference process, it is worthwhile to consider uncertainty about the preference process. Of course if we consider the world to consist of one agent whose preferences are the only factor that can drive prices it is hard to see how an individual would be uncertain concerning the driver of prices. However, thinking beyond a strict interpretation of the model, the drivers of housing preference may be unknown to any one agent and in a heterogeneous agent context it would be hard to know how aggregate housing preference would evolve. One can consider then the uncertainty about the preference process as standing in for the lack of heterogeneity in the model.

Beliefs

Firstly, I give a brief overview of how the agent's beliefs evolve. At the beginning of each period, the agent observes the current level of housing preference. Based on these data, he calculates a prediction error for two separate possible processes for the housing preference process, specifically a trend stationary model and a difference stationary model. Based on this prediction error, he updates the likelihood of each model and revises his beliefs according. Then based on these beliefs, he chooses demand for housing and the price of housing adjusts until demand equals supply. It is important to note that there is no impact of beliefs on the preference process for housing as this process is an exogenous process.

Now to explicitly calculate the parameters of each model of housing preference and the probability weights on the trend stationary and difference stationary model, I use the methods of Cogley and Sargent (2005). Their model uses Bayesian methods to recursively update the parameters on each model and then uses the likelihood of each model to calculate a probability weight on each model. For a given model (i.e., trend or difference stationary) indexed by $i = \{s, ns\}$, and a housing preference history Ξ^{t-1}, we assume that agent's prior beliefs about the model coefficients $\mathbf{\Theta_{i,t-1}}$ are distributed normally according to:

$$p(\mathbf{\Theta_{i,t-1}} | \sigma_i^2, \Xi^{t-1}) = N(\mathbf{\Theta_{i,t-1}}, \sigma_i^2 \mathbf{P_{t-1}^{-1}})$$

and their prior beliefs concerning the model residual variance are given by:

$$p(\sigma_{i,t-1}^2 | \Xi^{t-1}) = IG(s_{t-1}, v_{t-1})$$

Here N represents the normal distribution function, and IG represents the inverse-gamma distribution function. $\mathbf{P_{t-1}}$ is the precision matrix that captures the confidence the agent has in his belief for $\mathbf{\Theta_{i,t-1}}$ (the point estimates of the model coefficients), σ_i^2 is the estimate of the variance of the model residuals, s_{t-1} is an analogue to the sum of squared residuals, and v_{t-1} is a measure of the degrees of freedom to calculate the residual variance such that the point estimate of $\sigma_{i,t-1}^2$ is given by s_{t-1}/v_{t-1}. After observing aggregate housing preference ξ_t, the agent's posterior beliefs are given by:

$$p(\mathbf{\Theta_{i,t}} | \sigma_i^2, \Xi^t) = N(\mathbf{\Theta_{i,t}}, \sigma_i^2 \mathbf{P_t^{-1}})$$
$$p(\sigma_i^2 | \Xi^t) = IG(s_t, v_t)$$

Cogley and Sargent (2005) give the following recursion to update the parameters of the beliefs:

$$\begin{aligned}
\mathbf{P_t} &= \mathbf{P_{t-1}} + \mathbf{x_t}\mathbf{x_t'} \\
\mathbf{\Theta_t} &= \mathbf{P_t^{-1}}(\mathbf{P_{t-1}}\mathbf{\Theta_{t-1}} + \mathbf{x_t} y_t) \\
s_t &= s_{t-1} + y_t^2 + \mathbf{\Theta_{t-1}'}\mathbf{P_{t-1}}\mathbf{\Theta_{t-1}} - \mathbf{\Theta_t'}\mathbf{P_t}\mathbf{\Theta_t} \\
v_t &= v_{t-1} + 1
\end{aligned}$$

Here $\mathbf{x_t}$ is the vector of right-hand side variables for the model at time t and y_t is the left-hand side variable for the model at time t. This recursion gives the parameters of each model. Now it is necessary to calculate the probability weight on each model.

Given a set of model parameters: $\{\mathbf{\Theta_i}, \sigma_i\}$, we can calculate the conditional likelihood of the model as:

$$L(\mathbf{\Theta_i}, \sigma_i^2, \Xi^t) = \prod_{s=1}^{t} p(y_s|\mathbf{x_s}, \mathbf{\Theta_i}, \sigma_i^2)$$

where y_s and $\mathbf{x_s}$ are the left- and right-hand-side variables of the model at time s and Ξ^t is the housing preference history up to time t. Based on this likelihood, one can write the marginalized likelihood of the model by integrating over all possible parameters:

$$m_{it} = \iint L(\mathbf{\Theta_i}, \sigma_i^2, \Xi^t) p(\mathbf{\Theta_i}, \sigma_i^2) \mathrm{d}\mathbf{\Theta_i} \mathrm{d}\sigma_i^2$$

Then we have the probability of the model given the observed data $p(M_i|\Xi^t) \propto m_{i,t} p(M_i) \equiv w_{i,t}$. Here, we have defined the weight on model i, $w_{i,t}$ and $p(M_i)$ is the prior probability on model i.

Cogley and Sargent (2005) show that Bayes's rule implies

$$m_{it} = \frac{L(\mathbf{\Theta_i}, \sigma_i^2, \Xi^t) p(\mathbf{\Theta_i}, \sigma_i^2)}{p(\mathbf{\Theta_i}, \sigma_i^2|\Xi_t)}$$

and therefore:

$$\frac{w_{i,t+1}}{w_{i,t}} = \frac{m_{i,t+1}}{m_{i,t}} = p(y_{i,t+1}|\mathbf{x_{i,t}}, \mathbf{\Theta_i}, \sigma_i^2) \frac{p(\mathbf{\Theta_i}, \sigma_i^2|\Xi_t)}{p(\mathbf{\Theta_i}, \sigma_i^2|\Xi_{t+1})}. \tag{10}$$

We assume that regression residuals are normally distributed allowing us to use the normal p.d.f. to calculate $p(y_{i,t+1}|\mathbf{x_{i,t}}, \mathbf{\Theta_i}, \sigma_i^2)$. Cogley and Sargent (2005) show that $p(\mathbf{\Theta_i}, \sigma_i^2|\Xi_t)$ is given by the normal-inverse-gamma distribution and provide the analytical expressions for this probability distribution. Any choice of $\mathbf{\Theta_i}, \sigma_i^2$ will give the same ratio of weights; I use the posterior mean in my calculations.

This recursion implies the following recursion for model weights.

$$\frac{w_{s,t+1}}{w_{ns,t+1}} = \frac{m_{s,t+1}/m_{s,t}}{m_{ns,t+1}/m_{ns,t}} \frac{w_{s,t}}{w_{ns,t}} \tag{11}$$

Finally, to calculate the model probabilities, the consumer normalizes the weights to one, and therefore, the weight on the stationary model is given by:

$$p_{s,t} = \frac{1}{1 + w_{ns,t}/w_{s,t}}$$

Since housing preference shocks are an exogenous process, the model will eventually put all the weight on the true process. To allow for perpetual learning, I adapt the concept of constant gain learning from the least squares learning literature to the current setup.[15] I introduce a gain parameter (g) that overweights current observations. The gain probability can be interpreted as the probability of a structural break in the economy, such that the history of the housing preference process no longer has any bearing on the current process generating housing preferences; hence, the previous model weight ratio is set to one.

$$p_{s,t} = (1-g)\frac{1}{1+w_{ns,t}/w_{s,t}} + g\frac{1}{1+\frac{m_{s,t+1}/m_{s,t}}{m_{ns,t+1}/m_{ns,t}}} \tag{12}$$

Using the estimated probabilities, the equilibrium price from the learning model is then calculated as:

$$lnq_t^L = p_{s,t}lnq_t^S + (1-p_{s,t})lnq_t^{NS} \tag{13}$$

where lnp_t^S and lnp_t^{NS} represent the log prices calculated via Eqs. (6) and (7) where lnp_t^S is calculated using the stationary process (8) for housing preferences ξ_t and lnp_t^{NS} is calculated using the non-stationary process (9) for housing preferences.[16]

Calibration and Simulation

The model is tightly parameterized. Most calibrated parameters are standard, and I set them to standard values. Additionally, robustness to the parameter choices is shown in the "Robustness" section. Time is quarterly, and I set the discount rate in the model $\beta = 0.9975$. This implies a 3% annual real interest rate slightly higher than average rates on 10-year Treasury Inflation-Protected Securities (TIPS).[17] Low discount rates are consistent with the evidence in Giglio et al. (2015) who find very low discount rates when comparing the prices on housing with temporary ownership contracts versus permanent ownership contracts in the UK and Singapore. In the sticky price version of the model, I set $\lambda = 0.25$ to better match the positive autocorrelation of returns.[18] I set the lag length of the trend stationary and difference stationary model of housing preferences to 4.[19] I set $\gamma = 1$, a standard assumption, consistent with log utility.

I also need to calculate the prior beliefs of the agent; however, these do not matter much for the results because I simulate the model 500 times for 2000 periods

[15] See Evans and Honkapohja (2001).

[16] Importantly, I make a standard assumption from the learning literature that of anticipated utility (Kreps 1998), i.e., the agent makes decisions assuming his future beliefs will be the same as his current beliefs. This includes his beliefs about both the likelihood of each process and the implied covariances. However, beliefs can and do change in the future.

[17] The real interest rate in the model is given by $(1+g^c)^\gamma/\beta$ where g^c is the growth rate of consumption which is calibrated to be 0.5% per quarter.

[18] This calibration has one-quarter of houses updating their prices each quarter consistent with assumptions made in good markets, see Galí (2009).

[19] Four lag forecasting VARs are common in the literature. See, for example, Christiano et al. (1999) and Stock and Watson (2003, 2005).

each time and keep only the last $212 = (2013 - 1960) * 4$ observations to match the length of my data. The initial prior on the stationary model $p_{s,0} = 0.5$. To calibrate the initial beliefs for the preference processes ($\mathbf{\Theta_o}$), I estimate ordinary least squares regressions on the log housing rents series from Davis et al. (2008) deflated with the CPI. I assume that ε_t^s and ε_t^{ns} are distributed $N(0, \sigma_i^2)$ where σ_i^2 is estimated from the residuals in the previous estimation. I set the initial precision matrix $\mathbf{P_o} = 10^{-2} * \mathbf{I}$. This is a very defuse prior setting the standard error of the coefficients to 10 times the standard deviation of the regression residuals. I set s_o to the variance of the regression residuals and set the initial degrees of freedom (v_o) equal to 1. Finally, I assume that consumption growth is i.i.d., and estimate the process using real per-capita consumption of non-durables and services from the national income and product accounts. To estimate the covariance matrix $\mathbf{C}$ between the preference process shocks and the consumption shocks, I take the residuals from the preference process estimation and the consumption process estimation and calculate the variance-covariance matrix between these residuals. To simulate the stochastic shocks, I use a Cholesky decomposition of this variance-covariance matrix with the preference shock ordered first multiplied by a 2×1 vector of standard normal random variables. That is to say $[\varepsilon_t^i \varepsilon_t^c] = chol(\mathbf{C})' * [\varepsilon_t^1 \varepsilon_t^2]$ where ε_t^1 and $\varepsilon_t^2 \sim N(0, 1)$ for $i = \{s, ns\}$.

I set the gain parameter (g) equal to 0.005 which implies a structural break in the economy once every 50 years. Robustness to different values of the gain and a justification based on analysis of implied forecast errors are considered in the "Gain Justification and Forecast Errors" section.[20]

To evaluate the model, I assume the true preference process is the stationary process and simulate 500 trials of length 2000 keeping the last 212 observations to match the length of my data set.[21] I then report median statistics across the trials. Initial housing preferences are normalized to the steady-state value of the trend stationary model when t equals zero and consumption is initialized to be twice this value in line with US CPI data which suggest that housing represents 30% of the US consumption basket, though I find this initialization does not affect the results.

In addition to this being the most common assumption in the literature (Iacoviello 2010), I am motivated to make the true process a trend stationary process by a variety of concerns. The first is that the survey evidence outlined in the introduction suggests that individuals overreacted to the run-up in house prices and extrapolated current price changes far into the future. This evidence supports a true process for fundamentals being one with temporary deviations from trend and agents overreacting to these temporary deviations by assuming they are permanent. Additionally, in

[20] While the setup in this paper with model learning makes direct comparisons difficult my choice of gain appears to be on the low end of the choices in the literature. Least squares learning and Kalman gains in the literature range from 0.002 to 0.05. See, for example, Branch and Evans (2006), Eusepi and Preston (2011), Kuang and Mitra (2016) and Milani (2014).

[21] I choose the true process to be stationary for two reasons. First, it is the most common in the literature as explained in the "Housing Price" section. Second, I am interested in generating overreaction to fundamentals. A natural way to do this is to assume the shocks are temporary but agents mistake them as potentially permanent.

the US housing market temporary increases in prices may be persistent because of a slow response of supply. However, eventually supply can respond to bring prices down. A model where agents believe temporary shocks are permanent is consistent with neglecting the long-run supply response of housing.[22] Finally, Shiller (2005) and Reinhart and Rogoff (2009) argue that individuals often attribute new-era stories of fundamental change to justify high valuations or current booms as being permanent instead of temporary. This view of the world is consistent with my modeling. Of course, it would be possible to have the underlying process be a true switching process and agents form rational beliefs about what state they are in. However, I believe the spirit of that model is different than my goal in this paper. In that model agents are as likely to underreact as overreact. But here I try to capture the general notion that in speculative bubbles agents are overreacting in their long-run expectations and neglecting the tendency of fundamentals to return to long-run trends.[23]

To additionally motivate this belief structure, I compare this paper's methods with other potential approaches. The first is a Markov switching model where the preference process varies between the stationary and non-stationary formulations. Unfortunately, this approach does not generate significant variation in long-run forecasts. When transition probabilities are high, initial beliefs do not matter much for where you end up in the long run, when transition probabilities are low and you do not observe many in sample transitions. The second approach would allow for both permanent and transitory shocks. The agent could use the Kalman filter to learn. However, then the agent would react the same way to shocks (as a linear combination of permanent and transitory shocks with weights given by the shock's relative variances). This model would not generate endogenous time-varying volatility. A third approach would combine the two previous approaches: a switching model where in state one the preference shocks are permanent and in state two the preference shocks are only temporary. However, this model is not tractable. We would need to keep track of shocks and time-varying probabilities of all past states. My approach in this paper tries to capture the dynamics of this last approach in a straightforward, tractable way.

Results

In this section I compare results from the learning model with a rational expectations benchmark. The benchmark model is one in which the preference shock follows a stationary process and the household knows this. Results are reported for the flexible price model where $\lambda = 1$ so that prices immediately adjust to the fundamental value and then for $\lambda = 0.25$ so prices adjust more slowly.

[22] Fuster et al. (2012) argue that neglecting long-run mean reversion is a key psychological bias that is useful for understanding equity market puzzles.

[23] Finally, readers unhappy with the stationarity assumption should take comfort in the fact that the results will still hold if the preference process is difference stationary but still partially mean reverting and the agent puts some weight on a model where preferences are non-stationary with a smaller degree of mean reversion, see Tortorice (2018) and Fuster et al. (2012).

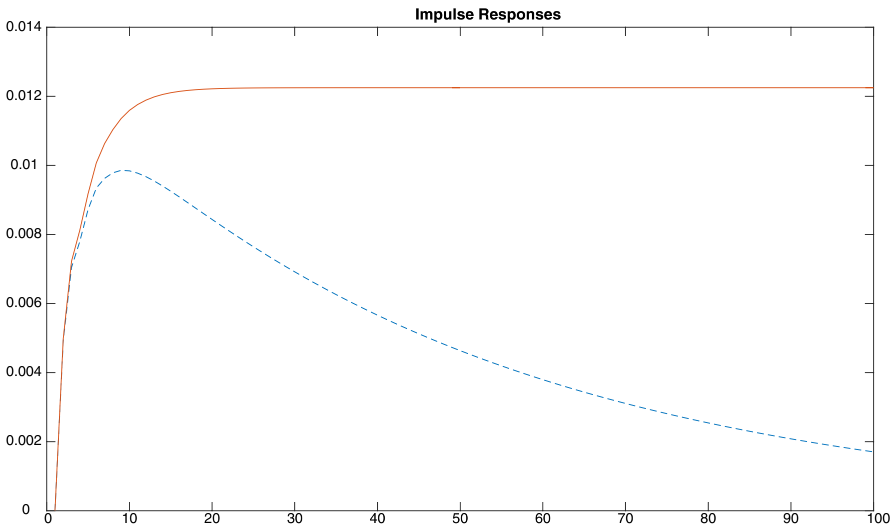

Fig. 1 Impulse responses: non-stationary model (solid), stationary (dashed)

Model Mechanism

Figure 1 plots impulse response functions for the preference processes [Eqs. (8), (9)] given a one standard deviation shock. On impact the effect on preferences is the same. One quarter out, both models also predict very similar effects on preferences because the short-term dynamics of the stationary model are very similar to the short-run dynamics of the non-stationary model. However, after the first few quarters the paths start to diverge with the non-stationary model predicting an increasing and permanent effect on preferences and with the stationary model predicting an effect that asymptotes to zero.

These dynamics have two important implication. First, when the agent believes in the non-stationary model, the price will be more volatile; each shock generates a large change in the present value of future fundamentals. However, if the agent believes in the stationary model, house prices will be smoother as shocks have only small effects on the present value. Second, the agent will be uncertain as to whether or not the stationary model is true. Note that the model predictions a few quarters out are not very different. Therefore, a random draw of the preferences generated by the stationary model will sometimes look as if they were generated by the non-stationary model. This result is consistent with the literature on unit roots in macroeconomic time series (Stock 1991; Deaton 1992; Cochrane 1988). These models are very hard to tell apart in time series the length of most macroeconomic series. Tests to distinguish between the two lack statistical power in small samples.

Now, to provide intuition for the main mechanism of the model, I examine a single simulated housing preference series and the implied path of rents and beliefs. Then, in the subsequent section I will provide median statistics over many

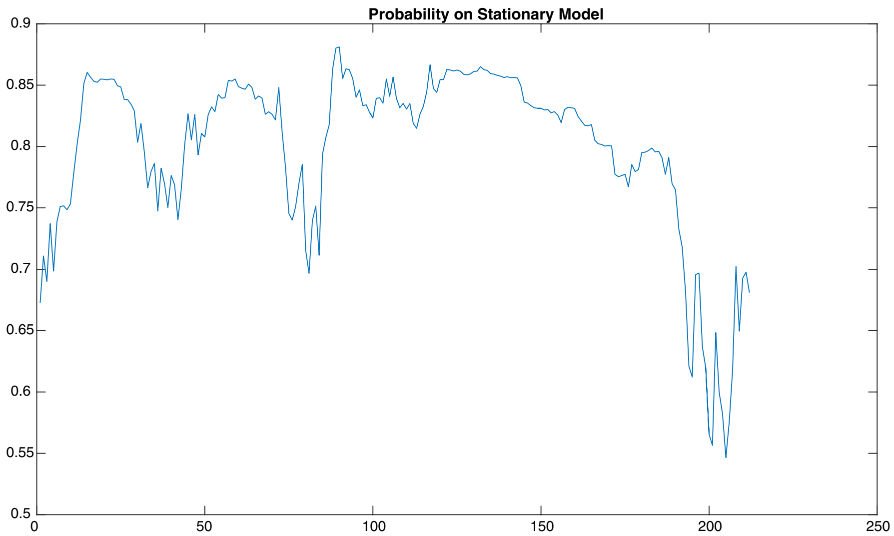

Fig. 2 Probability of the stationary model

simulations to evaluate the model. In Fig. 2, I plot the probability the learning model puts on the (trend) stationary model being true for a single housing preference series simulated from the (trend) stationary model. We see that on average the model puts more weight on the trend stationary model than the difference stationary model. However, this weight is not constant. Around time 175, we see the beliefs drift away from the trend stationary model where the agent goes from putting 75% weight on the stationary model to only putting 55% weight on the trend stationary model.

Recall that beliefs are endogenous here and depend on the realized housing preference and implied rent series.[24] Figure 3 plots the rent series that corresponds to the simulated housing preference series. At time 175, we can see the growth rate of rents increases resulting in a housing fundamental series that is persistently above trend.[25] Because the series is not reverting to trend, the agent begins to put more and more weight on the possibility that the housing preference series is difference stationary, revising his beliefs. Finally, around time 200 growth slows down, and the agent revises his beliefs, going back to putting about 70% of his weight on the trend stationary model.

Figure 4 plots the price-to-rent ratio under the learning model (dashed line) versus a rational expectations benchmark where the agent knows the true process is

[24] Recall that housing rents are given by the utility dividend from holding the housing asset.

[25] The exact date 175 does not have a particular significance. It is just important in this simulation because of the particular draw of random, exogenous rent shocks at this point. To confirm this explanation, I reran the simulation ten times recording the date of the peaks of the price-to-rent ratio in each simulation. I found that the peaks occurred at various times: 25, 200, 150, 80, 60, 45, 125, 110 190 and 140 to be exact.

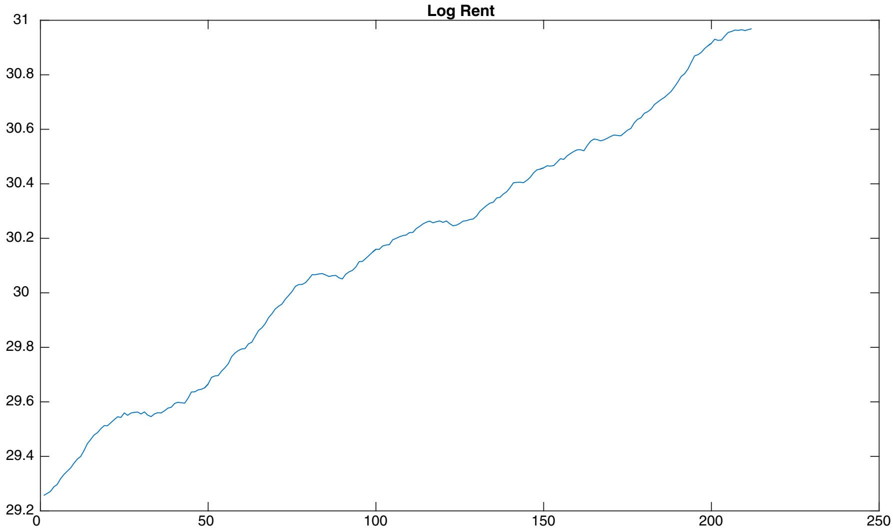

Fig. 3 Rent

trend stationary. We see a large spike in the price-to-rent ratio, increasing almost 20% relative to a slight fall under the rational expectations benchmark. There is a temporary housing boom while the agent believes that there has likely been a permanent increase in housing fundamentals which is then reversed with an abrupt fall in the price-to-rent ratio once the agent reverses his belief around time 200.

This mechanism is responsible for the main results of this paper. Agents overreact to news when the world looks as if it may be non-stationary. The overreaction is corrected once the fundamental begins to mean revert. This mechanism results in predictability of returns. Additionally, when the agent believes the world may be non-stationary, he reacts strongly to news resulting in a higher volatility of returns. These reactions generate time-varying volatility in returns.

Main Moments

Now I report median statistics for the model across the 500 simulations described in the "Calibration and Simulation" section. Results from the learning model and the rational expectations benchmark model compared with the main moments in the data are presented in Table 3. Examining the flexible price case first, note that both models imply a 3% annual average return on housing and a 3% growth rate of rent. In the data, these numbers are 6.4% and 1%, respectively.

The standard deviation of the log price-to-rent ratio $\sigma(\frac{P_t}{R_t}) = 15\%$ in the data. The rational expectations benchmark generates 1/5 this volatility, predicting a standard deviation of 3%. The learning model predicts 4%. The learning model also better matches the standard deviation of the HP-filtered price (P_t^{HP}) which was 3.7% in the data. The learning model predicts 3% versus only 0.8% for the rational expectations

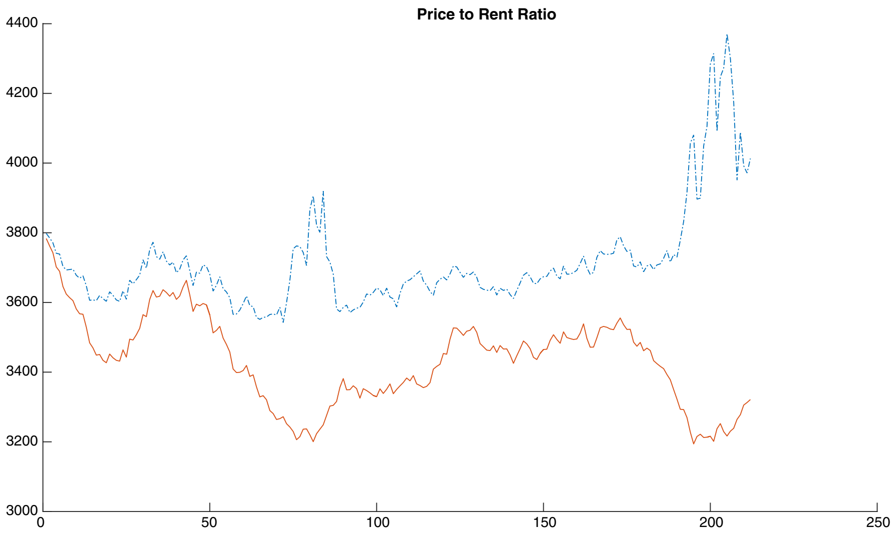

Fig. 4 Price-to-rent ratio: learning model (dashed), RE model (solid)

benchmark. I obtain a similar result for the standard deviation of returns, $\sigma(r_t)$, with the learning model predicting 8% versus the 6% in the data. The rational expectations benchmark model predicts only 3%.

The model's achievement in amplifying volatility is limited, especially regarding the volatility of the price-to-rent ratio. This should not be a complete surprise. The model is one in which prices are based on discounting fundamentals (rents) and the fundamentals are trend stationary. In this case, the present discounted value does not change very much over time because fundamentals always return to trend. Allowing agents to believe in the non-stationary model helps increase volatility, but the mechanism is limited since the true process is stationary and the agent's beliefs reflect a majority of the time. Additionally, generating volatility in the price-to-rent ratio is a demanding task when pricing is based on fundamentals. Since the only way prices can rise is if rents rise, even large increases in fundamentals may not change the price-to-rent ratio very much. Consequently, I now focus on moments where there is a better fit to the data, moments involving time-varying volatility and predictability.

Recall that both the price-to-rent ratio and returns are highly positively autocorrelated. Both models are consistent with the first fact; however, since the housing price is modeled as an asset price, neither model can explain the positive autocorrelation of returns.

Finally, I examine the ability of the model to explain the predictability of housing returns. The price-to-rent ratio is negatively correlated with future housing returns, $\rho(\frac{P_t}{R_t}, r_{t+1} + \cdots + r_{t+16}) = -0.74$. The rational expectations benchmark predicts a

Table 3 Model results: main moments

	Data	Flex price		Sticky price	
		RE	Learn	RE	Learn
Means, SD					
$\mathrm{E}(r_t)$	0.064	0.03	0.03	0.03	0.03
$\mathrm{E}(\Delta\ \ln(\mathrm{rent}_t))$	0.009	0.03	0.03	0.03	0.03
$\sigma(P_t/R_t)$	0.15	0.03	0.04	0.03	0.03
$\sigma(P_t^{\mathrm{HP}})$	0.037	0.008	0.03	0.005	0.02
$\sigma(r_t)$	0.06	0.03	0.08	0.01	0.04
$\sigma(\Delta\ \ln(\mathrm{rent}_t))$	0.023	0.03	0.03	0.03	0.03
Autocorrelations					
$\rho(P_t/R_t)$	0.99	0.99	0.9	0.97	0.96
	0.98	0.97	0.82	0.94	0.92
	0.95	0.94	0.76	0.91	0.88
	0.92	0.91	0.7	0.87	0.84
$\rho(r_t)$	0.84	0	− 0.01	0.74	0.44
	0.73	− 0.01	− 0.01	0.54	0.31
	0.66	0	− 0.01	0.4	0.22
	0.51	0	0	0.3	0.15
Predictability					
$\rho(P/R_t, r_{t+1} + \cdots r_{t+16})$	− 0.74	0.07	− 0.49	− 0.08	− 0.51
$\rho(r_t, r_{t+1} + \cdots r_{t+16})$	0.32	− 0.03	− 0.04	0.25	0.14
$\rho(P/R_t, \ln(\mathrm{rent}_{t+16}) - \ln(\mathrm{rent}_t))$	− 0.44	0.29	− 0.36	0.25	− 0.43
Campbell–Shiller regression					
β^{return}	− 0.8	0.06	− 1	− 0.06	− 1
R^2 (return regression)	0.5	0.04	0.24	0.04	0.27
$\beta^{\mathrm{rent\ growth}}$	− 0.12	0.46	− 0.42	0.37	− 0.5
R^2 (rent growth regression)	0.2	0.09	0.13	0.07	0.19

This table gives the model predictions for the main data moments

small positive correlation.[26] The learning model, however, predicts a correlation of − 0.49. Similarly, the price-to-rent ratio is negatively correlated with future rent growth $\rho(\frac{P_t}{R_t}, lnrent_{t+16} - lnrent_t)$. The learning model predicts a correlation coefficient of − 0.36 versus − 0.44 in the data. The rational expectations benchmark obtains the wrong sign for the correlation, predicting a substantial positive correlation. Finally, neither model can match the fact that the current return is positively correlated with future returns.

Campbell–Shiller style regressions tell a similar story to the raw correlations. The rational expectations model predicts no predictability of returns. The coefficient on returns $\beta^{\mathrm{return}} = 0.06$, and the $R^2 = 0.04$. In contrast, the learning model

[26] Small sample bias leads to a slight negative correlation instead of a value of zero.

generates a coefficient equal to −1 versus −0.8 in the data with an appreciable higher R^2 of 0.24. Similarly, the rational expectations model poorly explains rent growth predictability. The coefficient on rent growth in the Campbell–Shiller regression $\beta^{\text{rentgrowth}} = -0.12$; however, the rational expectations model predicts a coefficient equal to 0.46. However, the learning model matches the data much more closely. It predicts a coefficient of $\beta^{\text{rentgrowth}} = -0.42$ and an $R^2 = 0.13$.

To gain some intuition for these results, first note that as with most rational expectations models, only unpredictable changes in fundamentals affect future returns, so returns are unpredictable. On the other hand, the learning model generates return predictability. To understand the reason for return predictability in the learning model, note that in this model agents make expectational errors. Specifically, they think that the preference process could be non-stationary and so preferences might exhibit permanent deviations from trend. After a positive shock to preferences, agents expect the increase to be more permanent than it actually is. This error leads the price-to-rent ratio to increase relative to the rational expectations case. However, since preferences are actually trend stationary, they will not be permanently higher. When the agent sees these lower levels of housing preference in the future, the price falls resulting in lower returns. This mechanism creates a negative correlation between the price-to-rent ratio and future returns.

To understand the different results for rent growth predictability under rational expectations and learning, it is helpful to first consider the rational expectations case when the true process for rents is known. In this case rents are mean reverting, because housing preferences are mean reverting, and the agent knows this. Therefore, when rent is below trend, the agent expects rent to grow more quickly in the future to return to trend. As a result, the price which is the present discounted value of rents does not fall as much as rent does and the price-to-rent ratio will be high when rents are expected to grow more quickly in the future. However, in the learning model when rents fall, the agent considers the possibility that this could be a permanent fall in fundamentals. This lowers the price-to-rent ratio. Subsequently, though, rents increase because the actual process is stationary, creating a negative correlation between future rent growth and the current price-to-rent ratio.

Next, I examine the ability of the sticky price model to match the data, highlighting the differences relative to the flexible price model. Sticky prices lower the volatility of the HP-filtered price for the learning model, 2% versus 3% before, and returns 4% versus 8% as before. In both cases the learning model still increases volatility over the rational expectations benchmark.

Sticky prices have little effect on the autocorrelation predictions for the price-to-rent ratio. However, the model is now able to generate positive autocorrelation of returns. The rational expectations model predicts autocorrelations ranging from 0.74 at one lag to 0.3 at four lags versus 0.84 to 0.51 in the data. The learning model also predicts positive autocorrelation ranging from 0.44 to 0.15.

Finally, the rational expectations model predicts a small negative correlation between the price-to-rent ratio and future returns and a positive correlation between the price-to-rent ratio and rent growth. In the data these correlations are strongly negative. The learning model generates negative correlations consistent with the data. Indeed, it predicts a correlation between the price-to-rent ratio and future returns equal to − 0.51 versus − 0.74 in the data and between the price-to-rent ratio and future rent growth equal to − 0.43 versus − 0.44 in the data. Both models now are consistent with the positive correlation between the current return and future returns. The rational expectations model predicts 0.25, and the learning model predicts 0.14 versus 0.32 in the data.

Campbell–Shiller regressions for the sticky price model show a similar result. The rational expectations model again predicts no predictability of returns. The coefficient on returns $\beta^{\text{return}} = -0.06$, and the $R^2 = 0.04$. In contrast, the learning model generates a coefficient equal to -1 versus -0.8 in the data with an R^2 of 0.27. Similarly, the rational expectations model again does not explain rent growth predictability. The coefficient on rent growth $\beta^{\text{rentgrowth}} = -0.12$; however, the rational expectations model predicts a coefficient equal to 0.37. The learning model matches the data much more closely. It predicts a coefficient of $\beta^{\text{rentgrowth}} = -0.5$ and an $R^2 = 0.19$.

Time-Varying Volatility

Neither model can explain the skewness in the return data; however, the learning model generates positive skewness in the price-to-rent ratio. The learning model also generates higher kurtosis than the rational expectations benchmark. For the price-to-rent ratio, the learning model generates kurtosis equal to 4.4 versus 2.2 for the rational expectations benchmark. For returns, the learning model generates return kurtosis equal to 4.1 versus 2.9 for the rational expectations model. In the data kurtosis is about 7. In the sticky price case the learning model generates more kurtosis than the rational expectations model and comes close to matching the data on kurtosis of returns with a value of 5.1 (Table 4).

The flexible price results indicate that the rational expectations model does not generate any autocorrelation in squared returns $\rho(r_t^2, r_{t-1}^2)$ or residuals $\rho(\varepsilon_t^2, \epsilon_{t-1}^2)$. In contrast, model learning allows for endogenous time-varying volatility, though in the flexible price case the magnitudes of the correlations predicted by the model are smaller than in the data. The learning model predicts an autocorrelation of squared returns equal to 0.07 versus 0.5 in the data; the learning model predicts an autocorrelation of 0.09 on average for AR(1) return residuals versus 0.2 in the data. In the sticky price case, the rational expectations benchmark has autocorrelation in squared returns but not in the squared residuals. However, the learning model predicts positive autocorrelation for both squared returns and residuals. Additionally, for the squared residuals the magnitude predicted by the learning model is approximately correct, about 0.18 for the model versus 0.2 for the data.

Table 4 Model results: time-varying volatility

	Data	Flex price		Sticky price	
		RE	Learn	RE	Learn
Skewness and kurtosis					
skew(P_t/R_t)	2.04	0.02	0.89	0.05	0.68
skew(r_t)	− 1.7	0.03	0.13	− 0.03	0.15
kurtosis(P_t/R_t)	7.1	2.2	4.4	2.4	3.5
kurtosis(r_t)	7.4	3	4.1	2.9	5.1
Squared autocorrelations					
r_t^2	0.7	− 0.01	0.08	0.72	0.35
	0.56	0	0.07	0.53	0.27
	0.45	− 0.01	0.08	0.39	0.19
	0.27	− 0.01	0.06	0.28	0.15
$(\varepsilon_t^r)^2$	0.14	− 0.01	0.09	− 0.01	0.21
	0.39	− 0.01	0.1	− 0.01	0.19
	0.1	− 0.01	0.1	− 0.01	0.17
	0.32	− 0.01	0.08	− 0.01	0.17
GARCH model					
Υ_1 (GARCH)	0.73	0	0	0	0.68
	(0.06)				
a_1 (ARCH)	0.27	0	0	0	0.21
	(0.07)				
p value Engle test	0.04				

This table gives the model predictions for the key data moments

Results from estimating GARCH models on the simulated data give a similar result.[27] There is no evidence of GARCH effects in the rational expectations benchmark. However, we consistently find significant GARCH effects in the learning model data and of a similar magnitude to the data when we allow for sticky prices. For the sticky price model, the GARCH parameter equals 0.68 versus 0.73 in the data, while the ARCH parameter equals 0.21 versus 0.27 in the data. For the flexible price model, the median GARCH and ARCH parameters are zero. However, even with flexible prices, the learning model shows more evidence of GARCH effects. The Engle test for GARCH effects rejects 42% of the time for the learning model but only 7% of the time for the rational expectations model.

[27] To estimate the GARCH parameters, I first run an Engle test for the null hypothesis of no conditional heteroscedasticity on the simulated data. I estimate the GARCH parameters only if the test rejects, otherwise I assign zeros for the GARCH parameters. This procedure is required because of absent GARCH effects I am unable to identify the GARCH parameters using the maximum likelihood procedure.

Expected Returns

One of the important features of the model is that it generates predictability in housing returns without generating time-varying expected returns.[28] This is in contrast to rational expectations models with time-varying risk which generate predictability in returns which are expected by investors. Applying these models to the housing market, one would find that when the price-to-rent ratio is high investors would expect lower returns in the future. While these models are able to explain a negative correlation between the price-to-rent ratio and subsequent housing returns, they are at odds with an increasing large literature on survey expectations. For equity markets, survey results indicate that investors' expectations regarding future returns seem to be increasing in past stock market performance. As a result, high-price-to-earnings ratios are correlated, if anything, with higher expectations about future returns not lower (Fisher and Statman 2002; Shiller 2000; Greenwood and Shleifer 2014; Vissing-Jorgensen 2004). Similarly, in the housing market (Case and Shiller 2003; Piazzesi and Schneider 2009; Shiller 2007; Case et al. 2012), all find that expectations about future returns were increasing during the housing boom of the 2000s not declining. While I am unable to generate increasing expectations of future returns when the price-to-rent ratio rises, I am able to explain predictability in housing returns without time-varying expected returns. In this manner, my results are more in line with the survey evidence than models which require low expected returns when the price-to-rent ratio is high.

Robustness

Parameter Variation

The model has a small number of free parameters and therefore is straightforward to calibrate. However, I did set the AR lag length, the gain level g, the risk aversion coefficient γ, the discount factor β and the sticky price parameter λ. Table 5 gives the results from varying each of these parameters one at a time, while keeping the others at their original calibrated value. Except when varying the sticky price parameter λ, I use the sticky price model as the benchmark and set $\lambda = 0.25$. Table 5, therefore, demonstrates the robustness of the results to the various parameter choices.

There is little effect on the results of varying the AR lag length. I consider an AR length 2 and 6. We see that the learning model exhibits slightly more volatility when the AR lag length is 2 versus 4. For example, the standard deviation of the price-to-rent ratio is now 0.05 versus 0.03. The predictability correlations are all of the same sign and same magnitude under the alternative AR lag length calibrations as they are under the baseline calibration. The evidence of time-varying volatility remains;

[28] One-period-ahead expected returns are given by: $E_t[r_{t+1}] = E_t \frac{q_{t+1}}{q_t - d_t} = \exp[E_t(lnq_{t+1}) + \frac{1}{2}\sigma_q^2]/(q_t - d_t)$. This quantity is approximately constant in the model and equal to $(1 + g^c)^\gamma / \beta$ where g^c is the growth rate of consumption.

Table 5 Robustness

Moment	Data	AR	4	2	6	γ	1	2	3	β	0.9975	0.9891	0.9857
$\sigma(P_t/R_t)$	0.15		0.03	0.05	0.03		0.03	0.024	0.02		0.03	0.01	0.01
$\sigma(P_t^{\text{HP}})$	0.037		0.02	0.02	0.02		0.02	0.02	0.02		0.02	0.01	0.01
$\sigma(r_t)$	0.06		0.04	0.04	0.04		0.04	0.04	0.03		0.04	0.03	0.02
kurtosis(P_t/R_t)	7.1		3.5	2.6	3.5		3.5	3.4	3		3.5	2.9	2.9
kurtosis(r_t)	7.4		5.1	3.9	5.7		5.1	4.3	3.5		5.1	3.9	3.5
$\rho(P/R_t, r_{t+1} + \cdots r_{t+16})$	− 0.74		− 0.51	− 0.5	− 0.5		− 0.51	− 0.48	− 0.29		− 0.51	− 0.35	− 0.31
$\rho(r_t, r_{t+1} + \cdots r_{t+16})$	0.32		0.14	0.19	0.13		0.14	0.17	0.21		0.14	0.19	0.19
$\rho(P/R_t, \ln(\text{rent}_{t+16}) - \ln(\text{rent}_t))$	− 0.44		− 0.43	− 0.44	− 0.37		− 0.43	− 0.29	− 0.01		− 0.43	− 0.15	− 0.1
Autocorrelation: $(\varepsilon_t^r)^2$	0.14		0.21	0.14	0.21		0.21	0.18	0.14		0.21	0.13	0.12
	0.39		0.19	0.14	0.19		0.19	0.16	0.12		0.19	0.1	0.09
	0.1		0.17	0.12	0.17		0.17	0.15	0.11		0.17	0.09	0.09
	0.32		0.17	0.11	0.15		0.17	0.14	0.1		0.17	0.09	0.09

Moment	Data	g	0.005	0.001	0.01	λ	0.25	0.1	0.5
$\sigma(P_t/R_t)$	0.15		0.03	0.02	0.04		0.03	0.03	0.03
$\sigma(P_t^{\text{HP}})$	0.037		0.02	0.01	0.02		0.02	0.01	0.02
$\sigma(r_t)$	0.06		0.04	0.03	0.04		0.04	0.03	0.06
kurtosis(P_t/R_t)	7.1		3.5	2.9	3.4		3.5	2.9	4
kurtosis(r_t)	7.4		5.1	5.2	4.9		5.1	5.8	4.6
$\rho(P/R_t, r_{t+1} + \cdots r_{t+16})$	− 0.74		− 0.51	− 0.4	− 0.52		− 0.51	− 0.57	− 0.5
$\rho(r_t, r_{t+1} + \cdots r_{t+16})$	0.32		0.14	0.15	0.14		0.14	0.26	0.05
$\rho(P/R_t, \ln(\text{rent}_{t+16}) - \ln(\text{rent}_t))$	− 0.44		− 0.43	− 0.22	− 0.45		− 0.43	− 0.27	− 0.42
Autocorrelation: $(\varepsilon_t^r)^2$	0.14		0.21	0.21	0.19		0.21	0.23	0.14
	0.39		0.19	0.19	0.17		0.19	0.19	0.14
	0.1		0.17	0.17	0.18		0.17	0.17	0.13
	0.32		0.17	0.17	0.16		0.17	0.18	0.13

This table reports the robustness of the results from Table 3 when varying some of the parameters the autocorrelation of residuals from the AR(1) return regression is all positive and of similar magnitude.

Similarly increasing the gain from 0.005 to 0.01 slightly amplifies volatility of the price-to-rent ratio. Additionally, the predictability of rent growth increases slightly. The correlation of the price-to-rent ratio with future returns is − 0.52 versus − 0.51 for the baseline calibration. Similarly, the correlation of the price-to-rent ratio with future rent growth is equal to − 0.45 versus − 0.43 for the benchmark calibration. I also consider lowering the gain to 0.001. Here, volatility falls; for example, the standard deviation of returns is now 0.03 versus 0.04 in the benchmark calibration. And the model predicts less kurtosis of the price-to-rent ratio. However, the model still explains the kurtosis of returns, the predictability of returns and rent growth and the autocorrelation of squared returns.

Increasing γ from 1 to 3 has very little effect on the results. We see a slight fall in the level of volatility. At a level of $\gamma = 3$ the model exhibits a small degradation in its ability to explain kurtosis of returns, predictability of future rent growth and the autocorrelation of squared return residuals. However, it is still clear that the learning model improves over the benchmark rational expectations model in this case as well. Finally, increasing λ, the sticky price parameter, from 0.25 to 0.5 and reducing it from 0.25 to 0.1 have little effect on the results. We only see that with more flexible prices the model has slightly more difficulty explaining the predictability of future returns using current returns and the autocorrelation of square return residuals.

Finally, I consider additional calibrations of the discount factor β. The original calibration of β was chosen to be consistent with average real interest rates in the USA. In the robustness table I consider two additional ways to calibrate β. The first method is to match the expected return on housing in the data which leads to a value of $\beta = 0.9891$. The second method is to match the average rent-to-price ratio in the data which leads to a value of $\beta = 0.9857$.[29] I find that the moments for the different values of β are similar to the main moments reported in the table. The model generates less volatility than in the benchmark case, but it still generates higher volatility of returns and the HP-filtered house price than the rational expectations model. The model also generates excess kurtosis of returns. The price-to-rent ratio still predicts lower future returns with a correlation coefficient of −0.31 versus −0.51 in the benchmark case and predict lower future dividend growth with a correlation coefficient of −0.1 versus −0.43 in the benchmark case. We also still see positive autocorrelation of squared return residuals, though with a smaller effect. The coefficient on the first lag is 0.12 versus 0.21 in the benchmark case.[30]

[29] The steady-state expected quarterly return on housing is given by $E(r) = \frac{(1+g_c)^\gamma}{\beta}$, and the steady-state quarterly rent-to-price data is given by $\frac{rent}{P} = 1 - \beta\left[\frac{1+g_\xi+\gamma g_c}{(1+g_c)^\gamma}\right]$.

[30] In examining the robustness to β, I found it necessary to increase the flexibility of the estimation of the non-stationary model parameters so it did not converge to a model too close to the stationary model. To do so, I used a constant gain learning algorithm described in the appendix.

Table 6 Analysis of forecast errors

	Flex price		Sticky price	
	RE	Learn	RE	Learn
Rent				
$\rho(\varepsilon_t, \varepsilon_{t-1})$	– 0.0022	– 0.0049	– 0.0043	– 0.0052
$\sigma(\varepsilon_t)$	0.0072	0.0072	0.0072	0.0072
Returns				
$\rho(\varepsilon_t, \varepsilon_{t-1})$	– 0.0009	– 0.0071	– 0.004	– 0.007
$\sigma(\varepsilon_t)$	0.0269	0.0842	0.007	0.0348
$\sigma(\varepsilon_t)/\sigma(r_t)$	1	1	0.66	0.87

This table calculates autocorrelation and standard deviations for forecast errors under the learning model and the rational expectations benchmark model. The last row normalizes the standard deviation by the standard deviation of returns

Gain Justification and Forecast Errors

This subsection evaluates the magnitude of the expectational errors that agents make. First, note that the model is able to generate results using a very low gain value. For example, the gain value of 0.005 implies a structural break in the economy once every 50 years which seems eminently plausible. Additionally, it is worth noting that the model provides a direct check on the reasonableness of beliefs. Specifically, if the likelihood of the difference stationary model is low, the agent puts less weight on this model.

To further investigate the reasonableness of beliefs, Table 6 reports the median autocorrelation and standard deviation of one-step-ahead forecast errors across the model simulations for the model rent and return series. It compares the results from the rational expectations benchmark and the learning model. First, note that while the learning model implies a larger autocorrelation in forecast errors for the rent series, the number is still very small: – 0.005 versus – 0.002 for the rational expectations benchmark.[31] The results are similar for the sticky price model. Similarly, the standard deviation of forecast errors is the same across these two models: 0.72%, implying that the uncertainty does not generate large one-step-ahead forecast errors in the learning model.

Turing our attention to returns. The autocorrelation of forecast errors in the learning model is – 0.007 slightly larger than the value of – 0.001 for the rational expectations model. Given that both the learning model and the rational expectations benchmark are models of constant expected returns the standard deviation of return forecast errors just equals the unconditional standard deviation of returns. Again, results for the sticky price model are the essentially same, though the standard deviations fall as the volatility of returns falls.

These autocorrelation values are very small and would be impossible to distinguish from zero in reasonably sized samples. In contrast, one-step-ahead forecast errors for professional forecasters are often quite large with autocorrelation of 0.3

[31] Small sample bias prevents this correlation from being exactly zero.

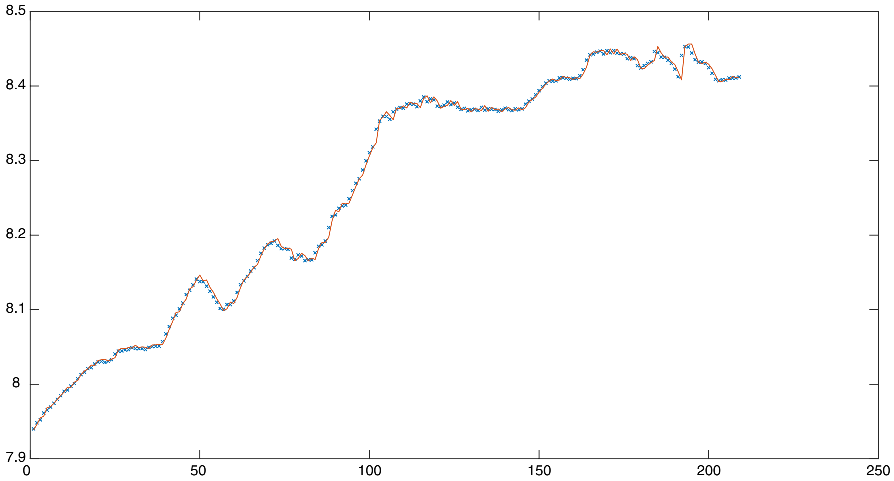

Fig. 5 Empirical fit of the log rent process: data (solid) and model (x)

or higher (Eusepi and Preston 2011) across many macroeconomic time series. Similarly, Caines (2015) finds an autocorrelation of house price return forecast errors equal to 0.4. Suggesting that this model, if anything, underestimates the degree of expectational errors. Additionally, the assumption of a constant expected return allows agents here to be better forecasters than surveys on US equity markets tend to indicate (Greenwood and Shleifer 2014), where agents expectations of returns rise when future returns are forecastably low.

Model Fit with Data

Most of the model results have relied on simulations of the model and attempts to match empirical moments in the data. Another interesting question is the extent to which the model can match the time series of rent and the price-to-rent ratio. The rent series in the model is an exogenous process. It is estimated based on the rent data and then the coefficients of the model are used to simulate a rent series that is then fed into the model to predict moments for housing returns, house prices and the price-to-rent ratio. Figure 5 examines the fit of the estimated rent model to the actual rent series. One can see that the fit is quite close and the estimated series tracks the actual series quite closely. To evaluate the fit of the model to the price-to-rent ratio, I simulate the model for 2000 periods. For the first $2000 - 212 = 1788$ periods, I use random shocks to minimize the effect of the prior choice on the model results. Then for the final 212 periods, I use the shocks from estimating the rent model on the data, i.e., the residuals from Fig. 5.

The results are summarized in Fig. 6. The model log price-to-rent ratio fluctuates in a way that is consistent with the price-to-rent ratio in the data. With the exception of early in the sample, it tends to rise when empirical measure rises and falls when

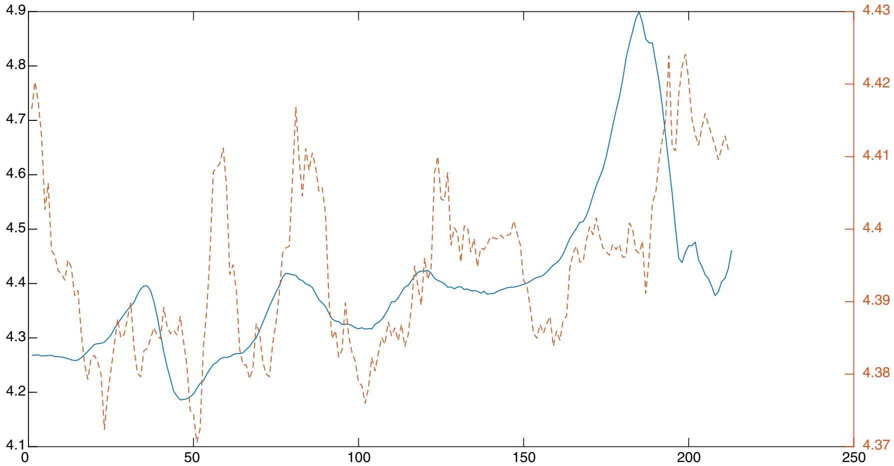

Fig. 6 Empirical fit of log price-to-rent ratio data (solid) and model (dashed)

the empirical measure falls. Note though the use of the secondary axis for the model log price-to-rent ratio. The fluctuations in the model are much smaller than in the data. This result is consistent with the low reported model price-to-rent ratio volatility compared to the data.[32]

Conclusion

Motivated by the large recent swing in US house prices and the dramatic impact, the housing crash had on real economic activity this paper has sought to explain key moments in the US macroeconomic time series on house prices and rents and specifically the role expectations may have played in generating these empirical facts. Given that the housing markets boom and bust was similar to booms and busts that have occurred in equity markets in the USA and beyond, we have focused on data moments that have received considerable attention in the analysis of equity markets.

The paper has documented that the price-to-rent ratio and housing returns are substantially more volatile than the underlying rent fundamentals. Both the price-to-rent ratio and housing returns exhibit momentum effects with strong positive autocorrelation in both the price-to-rent ratio and housing returns. Returns on housing are predictable with current returns forecasting higher returns in the future, while the price-to-rent ratio negatively forecasts both future returns and future rent growth. Finally, housing returns exhibit time-varying volatility as evidenced both my autocorrelation in squared returns and significant GARCH effects.

[32] Since the goal of this section is to examine the model fit to the price-to-rent ratio, I use the version of the model with β calibrated to match the price-to-rent ratio.

I show that a standard rational expectations benchmark is unable to match these facts. I then modify the standard model in two ways. I first allow for sticky prices so that house prices slowly adjust to their fundamental value. Then, I incorporate learning about the true nature of the housing preference process, specifically if the process is trend stationary (so shocks are temporary) or difference stationary (so shocks are permanent).

I find that these modifications improve the fit of the asset pricing model over the rational expectation benchmark. They amplify the volatility of prices and returns and explain the positive autocorrelation of returns. They also allow the model to explain the ability of the price-to-rent ratio to predict future returns and rent growth and help the model generate time-varying volatility similar to what is observed in the data.

This paper suggests that modeling expectations, particularly outside a strict rational expectations framework, is key to understanding the determinates of aggregate US house prices, especially in periods of booms and busts. Consequently, non-rational expectations should be incorporated into a wide variety of housing models and could significantly improve these models' fit with the data. Currently, many models explain large increases in asset prices with a decline in risk aversion. This paper implies that non-rational expectations mitigate the need to have risk aversion change over time and suggests that time variation in risk aversion may be overstated in previous research.

In addition to improving the fit to aggregate data and the realism of the underlying economic mechanism, there are several important policy implications of acknowledging the role overoptimistic expectations played in the housing boom. First, it provides a justification for monetary policy to "lean against the wind," i.e., raise interest rates to slow the growth of a potential housing bubble. Second, since overoptimistic expectations will lead to sub-optimal housing decisions by individuals, it suggests better financial literacy and numeracy is important for improving consumer welfare and macroeconomic outcomes. Finally, to the extent that non-rational expectations are present among financial institutions, the analysis of this paper supports macroprudential policies, like stress tests, that require large firms to consider how their firm would fair under adverse economic conditions that they do not view as especially likely.

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