Equity Return Predictability, Time Varying Volatility and Learning About the Permanence of Shocks

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Summary

Model

- Consumption based asset pricing model
- Shocks to fundamentals permanent or temporary?
- Use Bayesian learning with a constant gain to decide between these possibilities

Results

- Spikes and crashes in prices
- Predictability of returns
- Time varying volatility

Model – Representative Agent

$$\max \hat{E}_t \sum_{s=0}^{\infty} \delta^s \frac{-1}{\alpha} exp[-\alpha(c_{t+s} - \gamma c_{t+s-1})]$$

s.t.

$$b_{t+1} = Rb_t + c_t + (\Theta_t - \Theta_{t-1})p_t - \Theta_{t-1}d_t - y_t$$

- Safe asset return R
- Risky asset unit net supply, stochastic dividend

Model – What Dividend Process?

Trend Stationary (Temporary Shocks)

$$d_t = \alpha^s + \gamma^s t + \rho_1^s d_{t-1} + \ldots + \rho_p^s d_{t-p} + \varepsilon_t^s$$

OR

Difference Stationary (Permanent Shocks)

$$\Delta d_t = \alpha^{ns} + \rho_1^{ns} \Delta d_{t-1} + \ldots + \rho_p^{ns} \Delta d_{t-p} + \varepsilon_t^{ns}$$

Model – Learning:Parameters

Bayesian Linear Regression

- Given a dividend history D^{t-1}, parameter priors are:

$$p(\Theta_{i,t-1}|\sigma_i^2, D^{t-1}) = N(\Theta_{i,t-1}, \sigma_i^2 P_{t-1}^{-1})$$
$$p(\sigma_{i,t-1}^2|D^{t-1}) = IG(s_{t-1}, v_{t-1})$$

- Parameters of the beliefs satisfy the following recursion:

$$\begin{aligned} P_t &= P_{t-1} + x_t x_t' \\ \theta_t &= P_t^{-1}(P_{t-1}\theta_{t-1} + x_t y_t) \\ s_t &= s_{t-1} + y_t^2 + \theta_{t-1}' P_{t-1}\theta_{t-1} - \theta_t' P_t \theta_t \\ v_t &= v_{t-1} + 1 \end{aligned}$$

- Here x_t is the r.h.s variables and y_t is the l.h.s. variable

Model – Learning:Model Selection

Model Likelihood

$$m_{it} = \iint L(\Theta_i, \sigma_i^2, D^t) p(\Theta_i, \sigma_i^2) d\Theta_i d\sigma_i^2$$

Updating of model weights with a constant gain

$$\frac{w_{s,t+1}}{w_{ns,t+1}} = (1-g)\frac{m_{s,t+1}/m_{s,t}}{m_{ns,t+1}/m_{ns,t}}\frac{w_{s,t}}{w_{ns,t}} + g\frac{m_{s,t+1}/m_{s,t}}{m_{ns,t+1}/m_{ns,t}}$$

Model Probability

$$p_{s,t} = \frac{1}{1 + w_{ns,t}/w_{s,t}}$$

Model – Model Solution

Price

$$p_t = p_{s,t}\left[E_t \sum_{s=1}^{\infty} \frac{d_{t+1}}{R^s}|S\right] + (1-p_{s,t})\left[E_t \sum_{s=1}^{\infty} \frac{d_{t+1}}{R^s}|NS\right] - \Psi$$

Consumption

$$c_t = \frac{\gamma}{R}c_{t-1} + (1-\frac{\gamma}{R})\frac{R-1}{R}\left[-Rb_t + d_t + \hat{E}_t \sum_{s=1}^{\infty} \frac{d_{t+s}}{R^s}\right] - \Phi$$

Calibration and Data – Calibration

Parameters

- $R = 1.01$ (Annual)
- $\gamma = 0.7$ (Habit)
- $\alpha = 0$ (Risk Neutral)
- $g = 0.075$ (Gain needed for perpetual learning)

Simulation

- Assume true process is stationary.

Calibration and Data – Data

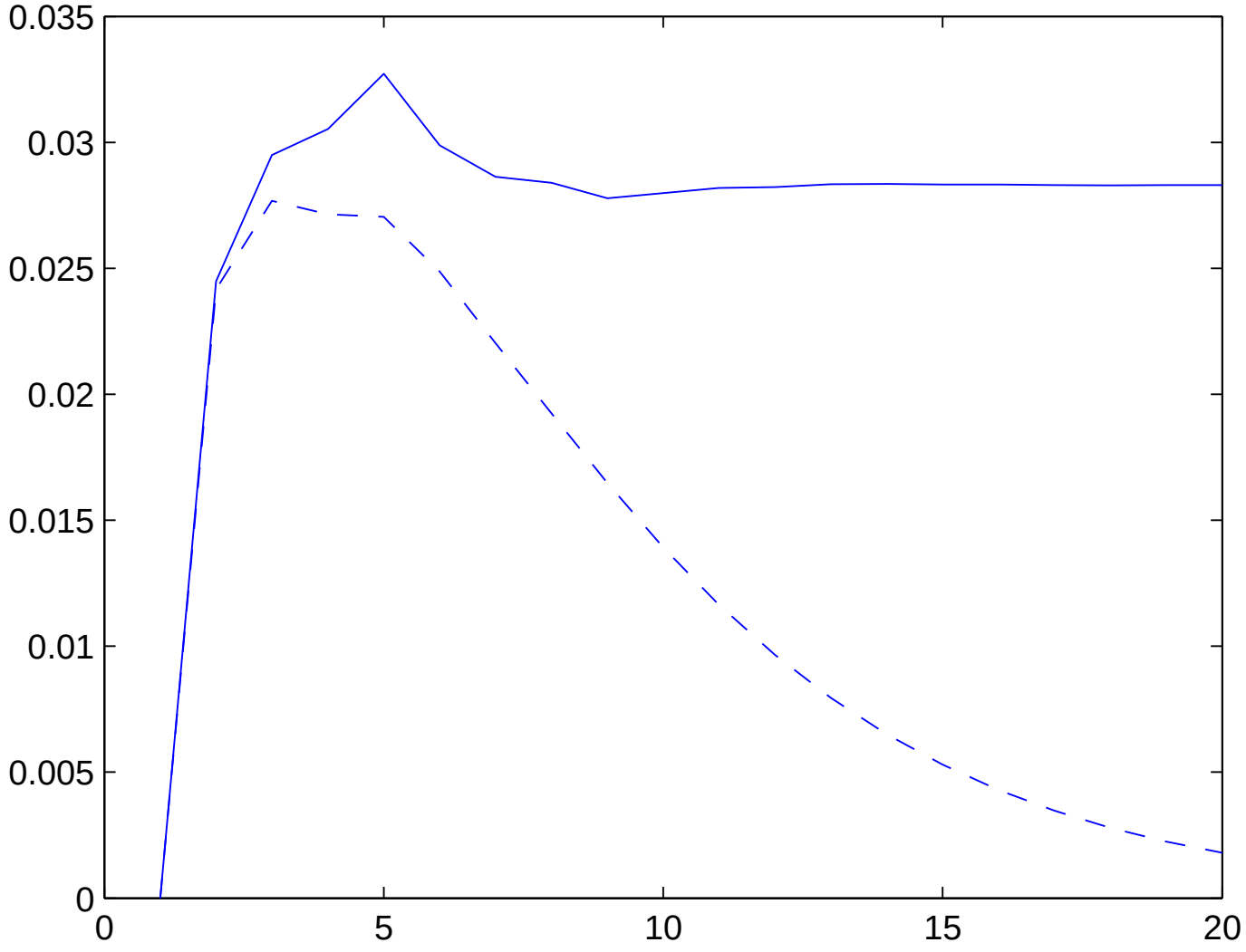
NIPA - Real Per-Capita Consumption

Shiller - PE-ratio, Dividends (S&P 500)

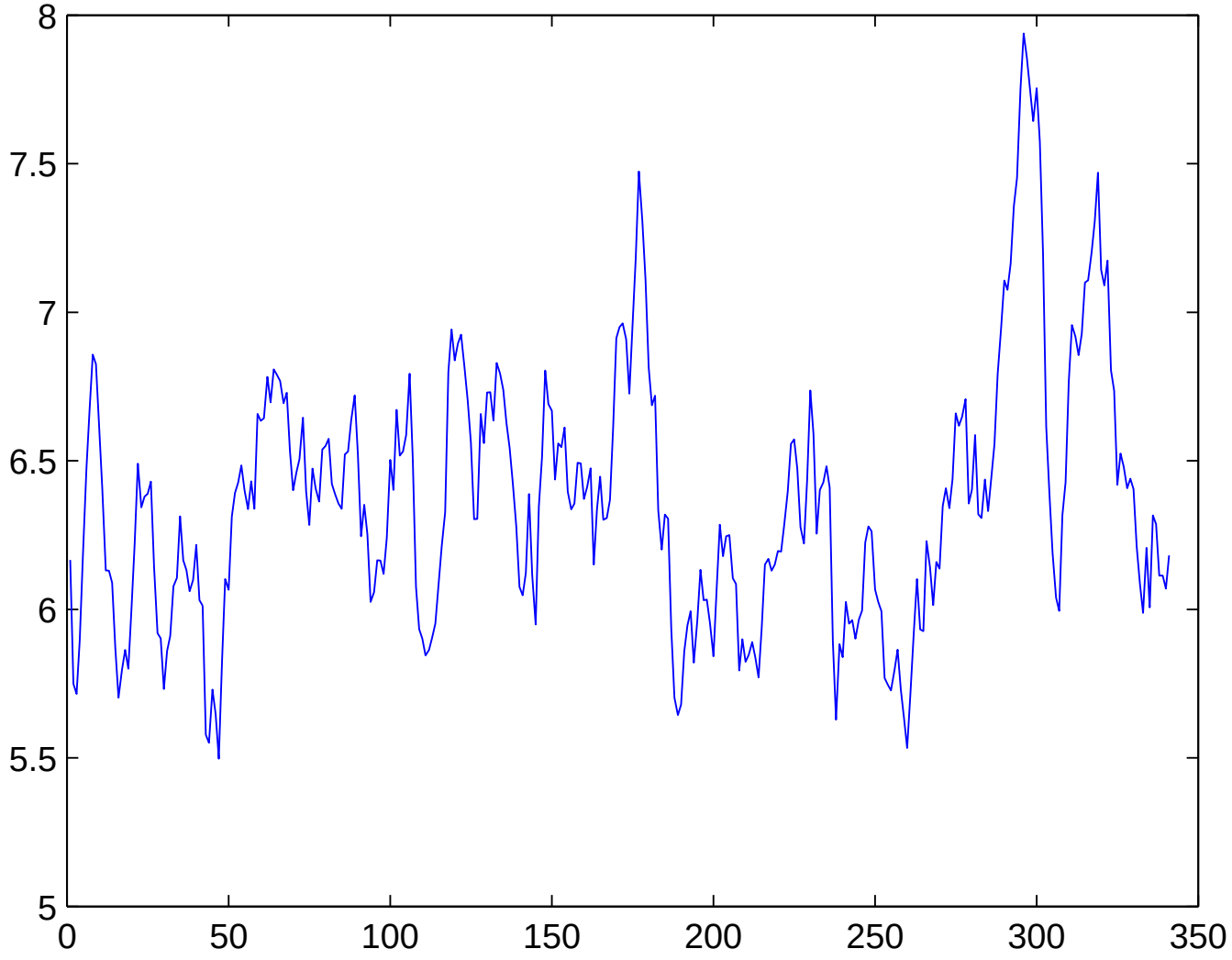
French - Excess return on market

Mechanism: Impulse Responses

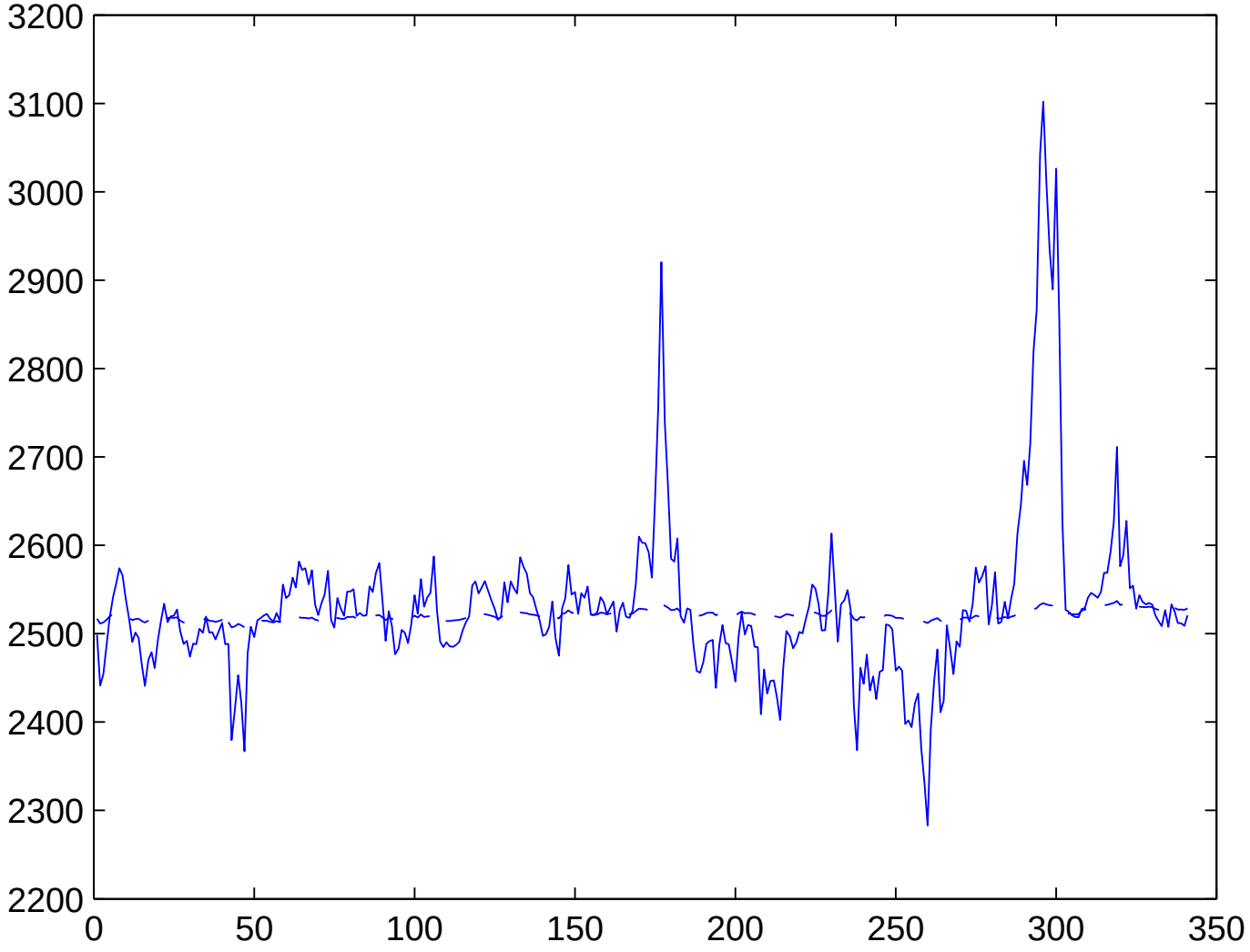
Non-Stationary vs Stationary (Dashed)



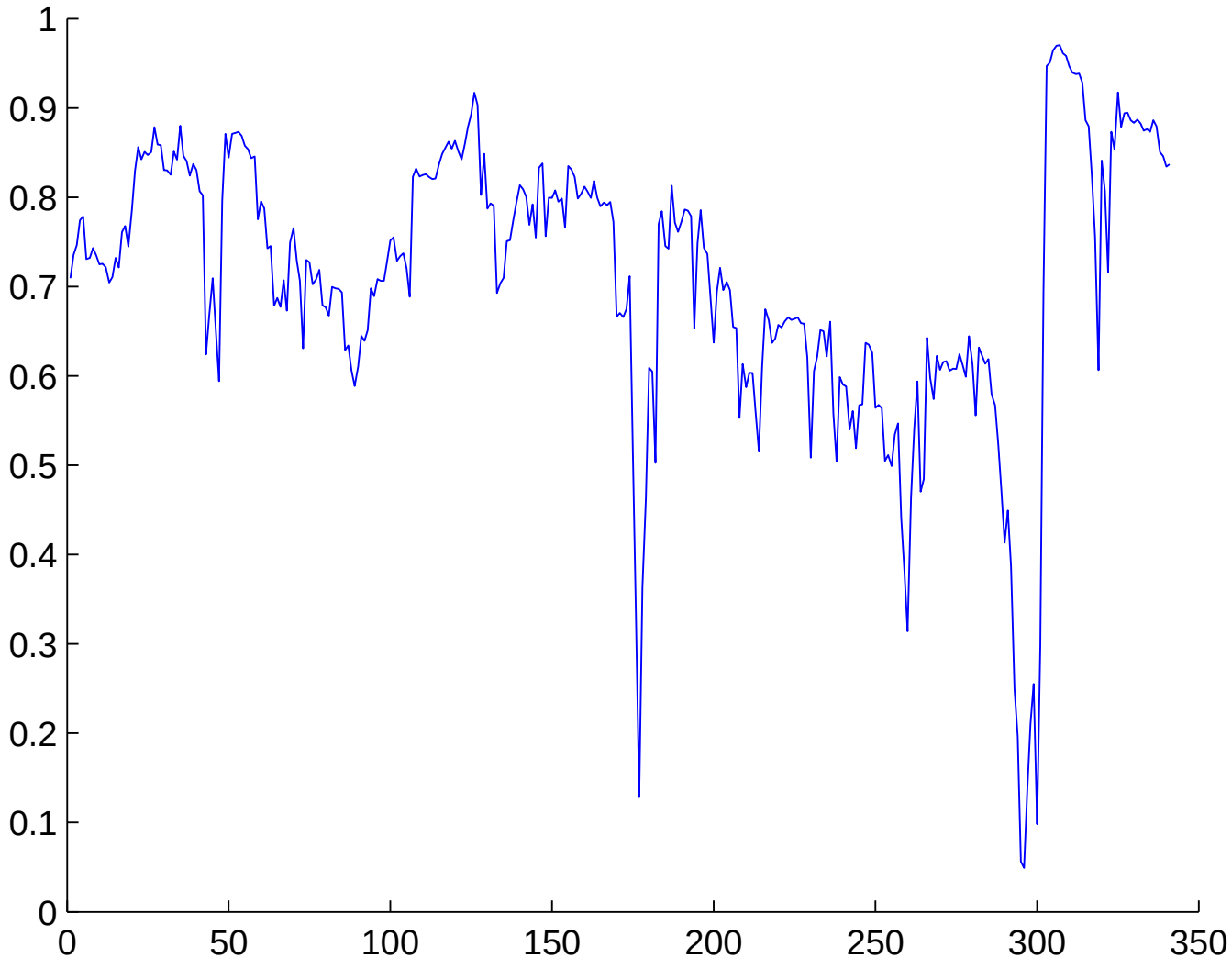
Mechanism – Dividend



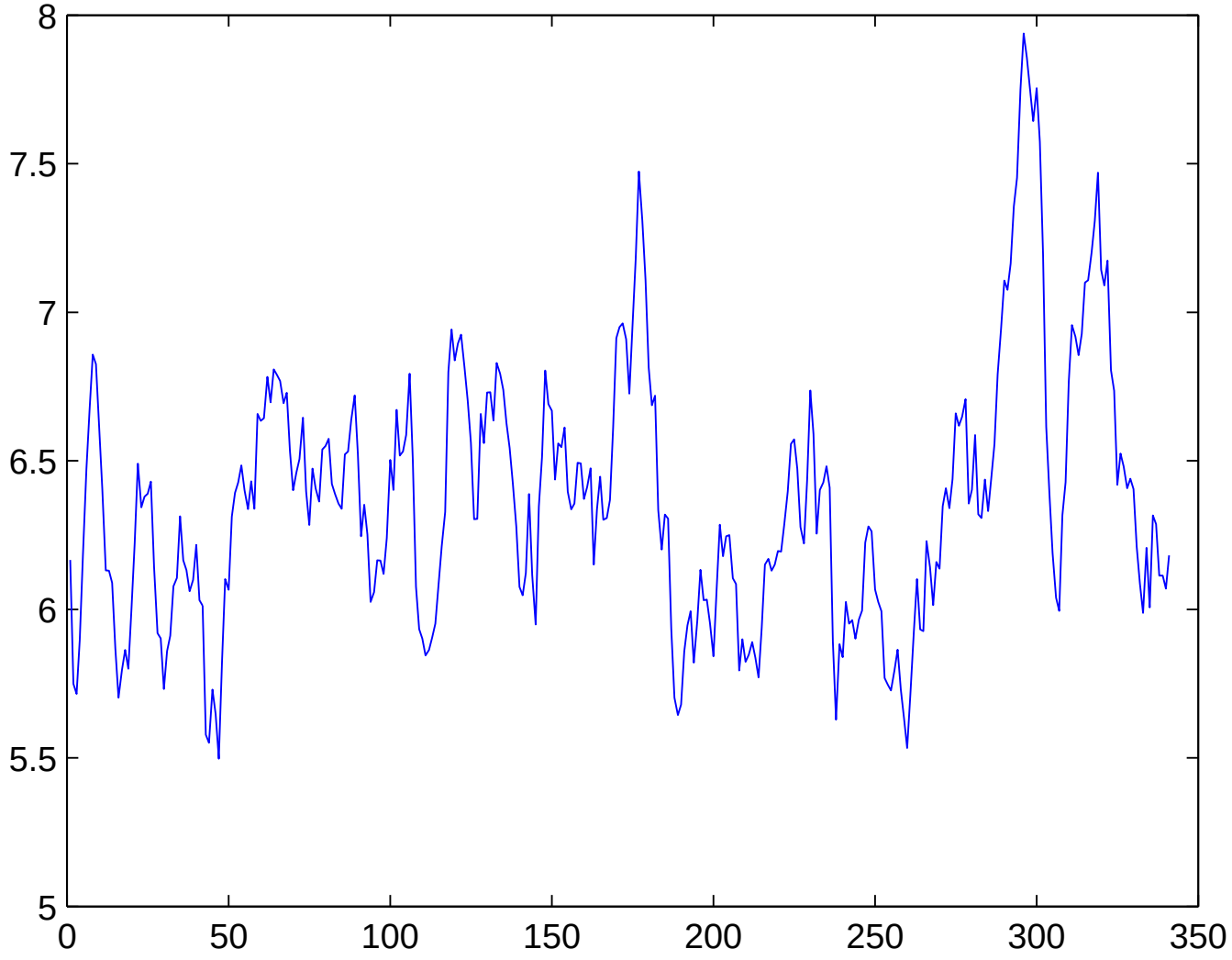
Mechanism-Price: Learning vs. RE



Mechanism – Prob. Stationary



Mechanism – Dividend



Results – Return Predictability

- Returns, PE and Consumption growth predict future returns.

Predicting Returns	Data	RE	Learning
$\mathrm{corr}(r_t, r_{t+2} + \ldots + r_{t+5})$	-0.2	-0.02	-0.21
$\mathrm{corr}(P/E_{10,t}, r_{t+2} + \ldots + r_{t+5})$	-0.41	0.12	-0.24
$\mathrm{corr}(\Delta \ln c_t, r_{t+2} + \ldots + r_{t+5})$	-0.34	-0.04	-0.26

Results – Cons. Predictability

- PE ratio and Consumption growth forecast consumption growth.

Predicting Consumption	Data	RE	Learning
$\mathrm{corr}(P/E_{10,t}, \Delta lnc_{t+3} + \ldots + \Delta lnc_{t+6})$	-0.16	0.13	-0.22
$\mathrm{corr}(\Delta lnc_t, \Delta lnc_{t+3} + \ldots + \Delta lnc_{t+6})$	-0.23	-0.03	-0.23

Results – Dividend Forecasts

- PE Ratio Negatively Forecasts Future Dividend Growth.

Predicting Dividends	Data	RE	Learning
$\mathrm{corr}(P/E_{10,t}, \Delta \ln d_{t+2} + \ldots + \Delta \ln d_{t+5})$	-0.25	0.16	-0.34

Results – Volatility

- Model amplifies return and consumption volatility

Volatility	Data	RE	Learning
$\sigma(r_t)$	20.50%	0.13%	3.2%
$\sigma(\Delta \ln c_t)$	2%	0.1%	2%

Results – Kurtosis

- Amplified Kurtosis

Kurtosis	Data	Confidence Bounds	RE	Learning
r_t	4.1	[2.58 3.46]	3.00	6.37
$P/E_{10,t}$	4.6	[1.67 3.50]	2.79	3.09
$\%\lvert r_t\rvert > 1.96*\sigma(r_t)$	6.2%	[3% 7%]	5.0%	5.6%

Results – Time Varying Volatility

- Positive autocorrelation of squared returns

Autocorrelation of Squared Returns

	Data	Standard Error	RE	Learning
lag 1	0.079	0.065	0	0.2
lag 2	0.01	0.065	-0.002	0.16
lag 3	0.47	0.065	0.001	0.13
lag 4	0.14	0.065	0.004	0.1

Results – Time Varying Volatility

- GARCH effects

GARCH	Data	Standard Error	RE	Learning
Garch	0.61	0.09	0	0.56
Arch	0.29	0.07	0	0.25
p-value Engle test	0.048			

Conclusion

- Consumption Based Asset Pricing Model
- Uncertainty about permanence of shocks.
- Predictability
- Time Varying Volatility