Equity Return Predictability, Time Varying Volatility and Learning About the Permanence of Shocks

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Summary

Model

- Consumption based asset pricing model
- Shocks to fundamentals permanent or temporary?
- Use Bayesian learning with a constant gain to decide between these possibilities

Results

- Spikes and crashes in prices
- Predictability of returns
- Time varying volatility
Model – Representative Agent

\[
\max \hat{E}_t \sum_{s=0}^{\infty} \delta^{s-1} \alpha \exp[-\alpha(c_{t+s} - \gamma c_{t+s-1})] \\
\text{s.t.}
\]

\[
b_{t+1} = Rb_t + c_t + (\Theta_t - \Theta_{t-1})p_t - \Theta_{t-1}d_t - y_t
\]

- Safe asset return \( R \)
- Risky asset unit net supply, stochastic dividend
Model – What Dividend Process?

Trend Stationary (Temporary Shocks)

\[ d_t = \alpha^s + \gamma^s t + \rho_1^s d_{t-1} + \ldots + \rho_p^s d_{t-p} + \varepsilon_t^s \]

OR

Difference Stationary (Permanent Shocks)

\[ \Delta d_t = \alpha^{ns} + \rho_1^{ns} \Delta d_{t-1} + \ldots + \rho_p^{ns} \Delta d_{t-p} + \varepsilon_t^{ns} \]
Model – Learning: Parameters

Bayesian Linear Regression

▶ Given a dividend history \( D^{t-1} \), parameter priors are:

\[
p(\Theta_{i,t-1} | \sigma^2_i, D^{t-1}) = N(\Theta_{i,t-1}, \sigma^2_i P_{t-1}^{-1})
\]
\[
p(\sigma^2_{i,t-1} | D^{t-1}) = IG(s_{t-1}, v_{t-1})
\]

▶ Parameters of the beliefs satisfy the following recursion:

\[
P_t = P_{t-1} + x_t x'_t
\]
\[
\theta_t = P_{t-1}^{-1} (P_{t-1} \theta_{t-1} + x_t y_t)
\]
\[
s_t = s_{t-1} + y_t^2 + \theta'_{t-1} P_{t-1} \theta_{t-1} - \theta'_t P_t \theta_t
\]
\[
v_t = v_{t-1} + 1
\]

▶ Here \( x_t \) is the r.h.s variables and \( y_t \) is the l.h.s. variable
Model Learning: Model Selection

Model Likelihood

\[ m_{it} = \int \int L(\Theta_i, \sigma_i^2, D^t) p(\Theta_i, \sigma_i^2) d\Theta_i d\sigma_i^2 \]

Updating of model weights with a constant gain

\[ \frac{w_{s,t+1}}{w_{ns,t+1}} = (1 - g) \frac{m_{s,t+1}/m_{s,t}}{m_{ns,t+1}/m_{ns,t}} \frac{w_{s,t}}{w_{ns,t}} + g \frac{m_{s,t+1}/m_{s,t}}{m_{ns,t+1}/m_{ns,t}} \]

Model Probability

\[ p_{s,t} = \frac{1}{1 + w_{ns,t}/w_{s,t}} \]
Model – Model Solution

Price

\[ p_t = p_{s,t} \left[ E_t \sum_{s=1}^{\infty} \frac{d_{t+1}}{R^s} |S| \right] + (1 - p_{s,t}) \left[ E_t \sum_{s=1}^{\infty} \frac{d_{t+1}}{R^s} |NS| \right] - \Psi \]

Consumption

\[ c_t = \frac{\gamma}{R} c_{t-1} + (1 - \frac{\gamma}{R}) R - 1 \left[ -Rb_t + d_t + \hat{E}_t \sum_{s=1}^{\infty} \frac{d_{t+s}}{R^s} \right] - \Phi \]
Calibration and Data – Calibration

Parameters

- \( R = 1.01 \) (Annual)
- \( \gamma = 0.7 \) (Habit)
- \( \alpha = 0 \) (Risk Neutral)
- \( g = 0.075 \) (Gain needed for perpetual learning)

Simulation

- Assume true process is stationary.
Calibration and Data – Data

NIPA - Real Per-Capita Consumption
Shiller - PE-ratio, Dividends (S&P 500)
French - Excess return on market
Mechanism: Impulse Responses
Non-Stationary vs Stationary (Dashed)
Mechanism – Dividend
Mechanism-Price: Learning vs. RE
Mechanism – Prob. Stationary
Mechanism – Dividend
Results – Return Predictability

- Returns, PE and Consumption growth predict future returns.

<table>
<thead>
<tr>
<th>Predicting Returns</th>
<th>Data</th>
<th>RE</th>
<th>Learning</th>
</tr>
</thead>
<tbody>
<tr>
<td>corr($r_t, r_{t+2} + \ldots + r_{t+5}$)</td>
<td>-0.2</td>
<td>-0.02</td>
<td>-0.21</td>
</tr>
<tr>
<td>corr($P/E_{10,t}, r_{t+2} + \ldots + r_{t+5}$)</td>
<td>-0.41</td>
<td>0.12</td>
<td>-0.24</td>
</tr>
<tr>
<td>corr($\Delta \text{ln}c_t, r_{t+2} + \ldots + r_{t+5}$)</td>
<td>-0.34</td>
<td>-0.04</td>
<td>-0.26</td>
</tr>
</tbody>
</table>
## Results – Cons. Predictability

- PE ratio and Consumption growth forecast consumption growth.

<table>
<thead>
<tr>
<th>Predicting Consumption</th>
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<th>RE</th>
<th>Learning</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\text{corr}(P/E_{10,t}, \Delta \text{ln}c_{t+3} + \ldots + \Delta \text{ln}c_{t+6}))</td>
<td>-0.16</td>
<td>0.13</td>
<td>-0.22</td>
</tr>
<tr>
<td>(\text{corr}(\Delta \text{ln}c_t, \Delta \text{ln}c_{t+3} + \ldots + \Delta \text{ln}c_{t+6}))</td>
<td>-0.23</td>
<td>-0.03</td>
<td>-0.23</td>
</tr>
</tbody>
</table>
Results – Dividend Forecasts

- PE Ratio Negatively Forecasts Future Dividend Growth.

<table>
<thead>
<tr>
<th>Predicting Dividends</th>
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<th>RE</th>
<th>Learning</th>
</tr>
</thead>
<tbody>
<tr>
<td>corr(P/E_{10,t}, \Delta \ln d_{t+2} + ... + \Delta \ln d_{t+5})</td>
<td>-0.25</td>
<td>0.16</td>
<td>-0.34</td>
</tr>
</tbody>
</table>
Results – Volatility

- Model amplifies return and consumption volatility

<table>
<thead>
<tr>
<th>Volatility</th>
<th>Data</th>
<th>RE</th>
<th>Learning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(r_t)$</td>
<td>20.50%</td>
<td>0.13%</td>
<td>3.2%</td>
</tr>
<tr>
<td>$\sigma(\Delta \text{ln} c_t)$</td>
<td>2%</td>
<td>0.1%</td>
<td>2%</td>
</tr>
</tbody>
</table>
## Results – Kurtosis

- **Amplified Kurtosis**

<table>
<thead>
<tr>
<th>Kurtosis</th>
<th>Data</th>
<th>Confidence Bounds</th>
<th>RE</th>
<th>Learning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_t$</td>
<td>4.1</td>
<td>[2.58 3.46]</td>
<td>3.00</td>
<td>6.37</td>
</tr>
<tr>
<td>$P/E_{10,t}$</td>
<td>4.6</td>
<td>[1.67 3.50]</td>
<td>2.79</td>
<td>3.09</td>
</tr>
<tr>
<td>$%</td>
<td>r_t</td>
<td>&gt; 1.96*\sigma(r_t)$</td>
<td>6.2%</td>
<td>[3% 7%]</td>
</tr>
</tbody>
</table>
Results – Time Varying Volatility

- Positive autocorrelation of squared returns

<table>
<thead>
<tr>
<th>Autocorrelation of Squared Returns</th>
<th>Data</th>
<th>Standard Error</th>
<th>RE</th>
<th>Learning</th>
</tr>
</thead>
<tbody>
<tr>
<td>lag 1</td>
<td>0.079</td>
<td>0.065</td>
<td>0</td>
<td>0.2</td>
</tr>
<tr>
<td>lag 2</td>
<td>0.01</td>
<td>0.065</td>
<td>-0.002</td>
<td>0.16</td>
</tr>
<tr>
<td>lag 3</td>
<td>0.47</td>
<td>0.065</td>
<td>0.001</td>
<td>0.13</td>
</tr>
<tr>
<td>lag 4</td>
<td>0.14</td>
<td>0.065</td>
<td>0.004</td>
<td>0.1</td>
</tr>
</tbody>
</table>
## Results – Time Varying Volatility

- **GARCH effects**

<table>
<thead>
<tr>
<th>GARCH</th>
<th>Data</th>
<th>Standard Error</th>
<th>RE</th>
<th>Learning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Garch</td>
<td>0.61</td>
<td>0.09</td>
<td>0</td>
<td>0.56</td>
</tr>
<tr>
<td>Arch</td>
<td>0.29</td>
<td>0.07</td>
<td>0</td>
<td>0.25</td>
</tr>
<tr>
<td>p-value Engle test</td>
<td>0.048</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Conclusion

- Consumption Based Asset Pricing Model
- Uncertainty about permanence of shocks.
- Predictability
- Time Varying Volatility