In today’s “world awash in numbers,” strong quantitative reasoning skills are required:

- in virtually all academic fields
- in most every profession
- and in decision-making in everyday life
  - in being an informed citizen
  - in making medical decisions
  - in making financial decisions

(Lynn Steen’s main argument in “The Case for Quantitative Literacy” in Mathematics and Democracy.)
QR is defined as:

- the ability to understand and use quantitative measures and inferences that allow one to function as a responsible citizen, productive worker, and discerning consumer. 
  (Bernie Madison)

- the ability to identify, understand and use quantitative arguments in everyday contexts. Quantitative literacy describes a habit of mind.
  (Deborah Hughes Hallett)
How does “QR” differ from “Math”? (Bernie Madison’s “two mathematics”)

Math
- Math track moves **vertically** to higher levels of abstraction….beauty and elegance
- Focus on **content** and components: algebra, geometry, statistics, calculus

QR
- QR reaches out **horizontally**, applying processes of reasoning, deduction, and analysis to a wide array of applications in many practical fields
- Focus on the hands-on use of logic, statistics, and math to solve problems in **context**-rich settings
QR Competencies
(MAA’s Standards for College Students)

• Reading and understanding quantitative info in graphs, tables, etc.
• Interpreting quantitative info and drawing appropriate inferences
• Solving problems using logic, math, statistics
• Estimating answers and checking for reasonableness
• Communicating quantitative info – verbally, graphically, numerically
• Recognizing the limitations of mathematical or statistical models
QR Fits Well at Holy Cross!

- QR is an essential component of College Readiness.
- Early assessment and coursework in QR can ensure students are prepared for introductory coursework in the STEM fields and in the quantitative social sciences.
- QR can also enhance students’ writing, when quantitative evidence enhances written arguments.
Background on QR at Other Colleges

- Wellesley College
- Bowdoin College
- Other colleges, including Hollins, JSU, Carleton
Wellesley College’s Two-part QR Requirement

1. **Basic skills requirement** —
   Pass QR Assessment (18 open-ended questions) or pass Basic Skills Course in first year

2. **QR overlay requirement** —
   Take course that emphasizes analysis of data. Note: Wellesley College also has a mathematical modeling requirement.
More on QR Basic Skills Course

- 8-10% of Wellesley’s entering class each year enrolls in QR 140, the basic skills course.
- Logic, math, basic stats taught in contexts, e.g. personal finance.
- Problem solving (esp. with real world, ill-structured problems); modeling; communication.
- PSs, tests, final project.
Wellesley College Offers
QR Overlay Courses in....

Astronomy
Biology
Chemistry
Computer Sciences
Geology
Mathematics
Physics
Economics
Education
Philosophy
Political Science
Psychology
and
Sociology
“Celebrating QR Connections”
Series at Wellesley College

QR & Art (Spring 2004)
QR & Biology (Fall 2005)
QR & Forensic Evidence (Spring 2007)
QR, Polling & Predictions (Fall 2008)
QR & Women’s Health (Spring 2010)
QR & Sustainability (Fall 2011)

Three or four lectures in each series — open to students, faculty, staff, the public…. 
Assessing How We’re Doing

- Student Attitude Assessment (NSF/Dartmouth). Four scales: utility, personal growth, ability, interest
- Pre- and Post- Assessments of Skills (post tests in QR 140 classes and exams)
- OIR analyses and surveys on students’ course-taking behavior and skills; senior surveys; alumnae surveys
- Continuous feedback from students & faculty
- Analysis of Students’ Writing (Carleton rubric)
- Regression Discontinuity Analysis
THE BOWDOIN QR PROGRAM

Holy Cross College
November 5, 2012
The QLR Assessment Project

- Combined Bowdoin, Colby-Sawyer, and Wellesley exams
- 10 Institutions and 1,659 students
- Mean 13.44 questions correct out of 23 (58.4%; sd = 5.35)
- Holy Cross (N = 652) Mean 14.38 (62.5%; sd = 4.31)
- LAS Schools (N = 1,011) Mean 15.28 (66.4%; sd = 4.60)

<table>
<thead>
<tr>
<th>Institution</th>
<th>Frequency</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bates</td>
<td>115</td>
<td>6.9</td>
</tr>
<tr>
<td>Bowdoin</td>
<td>100</td>
<td>6</td>
</tr>
<tr>
<td>Carleton</td>
<td>53</td>
<td>3.2</td>
</tr>
<tr>
<td>Colby-Sawyer</td>
<td>64</td>
<td>3.9</td>
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<tr>
<td>Central Washington</td>
<td>270</td>
<td>16.3</td>
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<td>652</td>
<td>39.3</td>
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<td>7.8</td>
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<tr>
<td>Southern Maine CC</td>
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<td>6.3</td>
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<tr>
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<td>91</td>
<td>5.5</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>1659</strong></td>
<td><strong>100</strong></td>
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</table>

The QLRA 13

- 13 questions identical to the Bowdoin Q-exam questions
  - #2, 7, 8, 9, 13, 15, 16, 17, 18, 19, 21, 22, 23
- Bowdoin 2012 Q-exam 30 questions (N = 497)

<table>
<thead>
<tr>
<th></th>
<th>Correl</th>
<th>Mean QLRA 13</th>
<th>STDEV QLRA 13</th>
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<tbody>
<tr>
<td><strong>Total</strong></td>
<td>0.959</td>
<td>6.82</td>
<td>3.315</td>
</tr>
<tr>
<td><strong>HC</strong></td>
<td>0.940</td>
<td>7.27</td>
<td>2.78</td>
</tr>
<tr>
<td><strong>Bowdoin</strong></td>
<td>0.913</td>
<td>9.02</td>
<td>2.72</td>
</tr>
</tbody>
</table>
The QLRA 13 Distribution

- ~40% of HC students scored below 50%
  - Under 50% on Bowdoin Q-exam criteria for Math 050
- HS Calculus (p<0.001) at Holy Cross

<table>
<thead>
<tr>
<th>Course</th>
<th>N</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Mean</th>
<th>Std. Deviation</th>
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<tr>
<td>Yes HS Calculus</td>
<td>403</td>
<td>5</td>
<td>23</td>
<td>15.47</td>
<td>3.883</td>
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</table>

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>0.959</td>
<td>6.82</td>
<td>52.5%</td>
<td>3.315</td>
<td>25.5%</td>
</tr>
<tr>
<td>HC</td>
<td>0.940</td>
<td>7.27</td>
<td>55.9%</td>
<td>2.78</td>
<td>21.4%</td>
</tr>
<tr>
<td>Bowdoin</td>
<td>0.913</td>
<td>9.02</td>
<td>69.4%</td>
<td>2.72</td>
<td>20.9%</td>
</tr>
</tbody>
</table>
The Bowdoin QR Exam

- 30 question entrance exam used for advising
  - Under 50% on Bowdoin Q-exam criteria for Math 050 (N = 50)

- Lessons Learned
  - Replace procedural, algorithmic questions with more involved reasoning, critical thinking questions.
  - Ask students to interpret tables and charts rather than doing it for them.
  - Focus on quantitative literacy, using numbers in meaningful sentences rather than just computation.
  - Ask students to postulate possible explanations for statistics rather than traditional logic games.
The Bowdoin QR Exam

- 30 question entrance exam used for advising
  - Under 50% on Bowdoin Q-exam criteria for Math 050 (N = 50)
- Significant predictor of GPA (N = 3,000)
  - Cumulative GPA  \( r = 0.39 \)
  - MCSR GPA  \( r = 0.48 \)
- Strongly correlated with 1st year Cum GPA  \( r = 0.48 \)
- Multivariate Regression Models ( \( R^2 = 0.30 \) Cum GPA and \( R^2 = 0.36 \) MCSR GPA)
The Bowdoin QR Exam

- Multivariate Regression Models ($R^2 = 0.30$ Cum GPA and $R^2 = 0.36$ MCSR GPA)

- These coefficients indicate the predicted difference in GPA associated with a 10 percentage point increase in respective aptitude test, with all other variables in model held constant.

<table>
<thead>
<tr>
<th>Cumulative GPA Multivariate Regression Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math SAT</td>
</tr>
<tr>
<td>0.0345</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MCSR GPA Multivariate Regression Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math SAT</td>
</tr>
<tr>
<td>0.1711</td>
</tr>
</tbody>
</table>
“Quantitatively literate citizens need to know more than formulas and equations. They need a predisposition to look at the world through mathematical eyes, to see the benefits (and risks) of thinking quantitatively about commonplace issues, and to approach complex problems with confidence in the value of careful reasoning. Quantitative literacy empowers people by giving them tools to think for themselves, to ask intelligent questions of experts, and to confront authority confidently. These are skills required to thrive in the modern world.”

- Mathematics and Democracy 2001

In short, how do we create a mathematics curriculum which teaches our students how to THINK?
“A typical American school day finds some six million high school students and two million college freshman struggling with algebra.”

“I’m not talking about quantitative skills, critical for informed citizenship and personal finance, but a very different ballgame.”

“What is needed is not textbook formulas but greater understanding of where various numbers come from and what they actually convey.”
This is not a pipeline... it is a trickle. 60% of STEM workforce is 45 and older.
How Much Math Do We Really Need?

- Professor Emeritus U. Ill. Chicago
  Washington Post 10/22/2010

“Unlike literature, history, politics and music, math has little relevance to everyday life.”

“All the math one needs in real life can be learned in early years without much fuss.”

“Most adults have no contact with math at work, nor do they curl up with an algebra book for relaxation.”
“Few in business today make use of the mathematics they learned in school. **Spreadsheets** have entirely different requirements.”

“Make no mistake; the revolution in business math created by the spreadsheet is conceptual as well as physical. **It changes the way people in business think** about and approach problems as well as the way they work through results. It enables them to quantify a whole new range of problems.”

“Mathematical reasoning in workplaces **differs markedly** from the algorithms taught in school.”
Problem Solving vs. Modeling

- Modeling for Insight

Powell and Batt

- Well Structured Problems
  - Objective Clear
  - Assumptions Obvious
  - Data available
  - One right answer

- Ill Structured Problems
  - Objectives, Assumptions, Data ambiguous

- Examples
  - Solve $2x - 5/x = 12$ for $x$.
  - Balance the books.
  - Do your taxes.
  - Seriously, do your taxes!
  - Should the Red Cross pay for blood donations?
  - Should we tax soda?
  - How much should an advertiser allocate to creative over delivery of ad?
  - Should spreadsheets be taught K-12?
Ill structured problems are *explored* not solved.

**Modeling Process**
- Frame the Problem
- Diagram the Problem
- Influence Diagrams (relationships between variables)
- Build a Model
- Spreadsheet Engineering/ Parametrization
- Sensitivity/ Strategy Analysis
- Generate Insights
- Iterate!

- **Amortization**
  - Buy versus Rent Scenario
  - Make Assumptions, Formulate Hypotheses, Generate Insights (don’t “solve!”)

- **Evaluating Colleges**
  - “Use data like tuition, acceptance rates and faculty information to rank schools.”

- **Calculating Car Costs**
  - “Create scatter-plot of used car year and price.”

- **Metro Card Math**
  - Unlimited card or ride by ride?

- **Olympic Algebra**
  - “Compare and contrast average speeds of athletes over time, across events, and by gender.”

- **Stock Portfolios**
  - Compound interest formula, exponential growth, and compare different rates of return.
Ratio and Proportion

E = 4/3*D

Or is it
E = 3/4*D ??
### Modeling Car Cost

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Focus</th>
<th>Prius</th>
<th>Cost</th>
<th>Gallons per 100 miles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$20,000.00</td>
<td>$33,000.00</td>
</tr>
<tr>
<td><strong>Gallons per 100 miles</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MPG Hway</td>
<td>28</td>
<td>45</td>
<td>3.571</td>
<td>2.222</td>
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<tr>
<td>MPG City</td>
<td>22</td>
<td>37</td>
<td>4.545</td>
<td>2.703</td>
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<tr>
<td><strong>Gallons Used</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Miles Hway</td>
<td>8,000</td>
<td></td>
<td>285.71</td>
<td>177.78</td>
</tr>
<tr>
<td>Miles City</td>
<td>12,000</td>
<td></td>
<td>545.45</td>
<td>324.32</td>
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<tr>
<td>Price per Gallon</td>
<td>$3.86</td>
<td></td>
<td>831.17</td>
<td>502.10</td>
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<tr>
<td><strong>Total Gallons</strong></td>
<td></td>
<td></td>
<td>$3,208.31</td>
<td>$1,938.11</td>
</tr>
<tr>
<td><strong>Cost</strong></td>
<td></td>
<td></td>
<td>$1,270.20</td>
<td>$1,938.11</td>
</tr>
<tr>
<td><strong>Gas Savings per year</strong></td>
<td>$1,270.20</td>
<td></td>
<td></td>
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<tr>
<td><strong>Extra Cost</strong></td>
<td>$13,000.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Years to recoup</strong></td>
<td>10.23</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Is Algebra Necessary?**
Yes! And we can use spreadsheets and modeling to help teach students why.

### Tornado Sensitivity Chart

![Tornado Sensitivity Chart](chart.png)
"41.1% of blacks were arrested in 1997, which means 7.4 out of every 1,000 people was a violent black criminal…"

Really? So 56.8% of whites were arrested for violent crimes as well?...

<table>
<thead>
<tr>
<th></th>
<th>Number</th>
<th>Percent</th>
<th>Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Victimization</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>White</td>
<td>7,068,590</td>
<td>82.1</td>
<td>37.1</td>
</tr>
<tr>
<td>Black</td>
<td>1,306,810</td>
<td>15.2</td>
<td>46.8</td>
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<tr>
<td><strong>Arrests</strong></td>
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<tr>
<td>White</td>
<td>284,523</td>
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</tr>
<tr>
<td>Black</td>
<td>205,823</td>
<td>41.1</td>
<td>7.4</td>
</tr>
</tbody>
</table>

Quantitative Literacy: Communicating (Reading and Writing) with Numbers NOT just Arithmetic
Women are 68% percent more likely than men to experience depression in their lifetimes.

Over 75% of women never experience depression in their lifetime.

17.1 percent of individuals have experienced depression in their lifetime.

Over 1 in 5 women and 1 in 8 men have experienced depression in their lifetimes.

Approximately four of every ten depressed individuals is a man.

Statistics: 21.3% of women and 12.7% of men have experienced depression in their lifetime.

“In other words, translating a ratio to a percentage is not just a mathematical operation, but also a rhetorical practice in which artistic appeals are manipulated.” - Joanna Wolfe
## Math 050: Quantitative Reasoning

### Pre-Post Assessments

#### Math 50: QR Spring 2011

<table>
<thead>
<tr>
<th></th>
<th>Pre-Q-Zscore</th>
<th>Post-Q-Zscore</th>
<th>Total Improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>-1.219</td>
<td>-0.253</td>
<td>0.966</td>
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<tr>
<td><strong>StDev</strong></td>
<td>0.905</td>
<td>0.913</td>
<td></td>
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</tbody>
</table>

#### Math 50: QR Fall 2011

<table>
<thead>
<tr>
<th></th>
<th>Pre-Q-Zscore</th>
<th>Post-QZscore</th>
<th>Total Improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>-1.337</td>
<td>-0.21</td>
<td>1.127</td>
</tr>
<tr>
<td><strong>StDev</strong></td>
<td>0.67</td>
<td>0.913</td>
<td></td>
</tr>
</tbody>
</table>
Bowdoin Quantitative Reasoning Program

- **Assessments**
  - CAT Test
  - Carleton Rubric:
    - 58 Papers, ~½ Q-relevant but only 5 Good/Exemplary Use of Q
  - Senior Survey Question (N = 370)
    - 77.8% No Mention of Data (7.6% (N = 28) meaningful argument)
- **Embedding Questions on Final Exams in MCSR Courses**

- **Tutoring**

- **Faculty Development**

---

### Table: Tutoring Hours

<table>
<thead>
<tr>
<th>School Year</th>
<th>No. of Study Groups (Fall)</th>
<th>No. of Study Groups (Spring)</th>
<th>No. of Individual Tutorials (Fall)</th>
<th>No. of Individual Tutorials (Spring)</th>
<th>No. of Tutors (Fall)</th>
<th>No. of Tutors (Spring)</th>
<th>No. of Tutor Hours (Total)</th>
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<tbody>
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<td>2012-13</td>
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<td>77</td>
<td></td>
<td></td>
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<tr>
<td>2011-12</td>
<td>39</td>
<td>35</td>
<td>32</td>
<td>20</td>
<td>88</td>
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<td>33</td>
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<td>47</td>
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<td>2008-09</td>
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<td>2007-08</td>
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<td>72</td>
<td>51</td>
<td>62</td>
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<td>48</td>
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<td>33</td>
<td>26</td>
<td></td>
<td></td>
<td>860</td>
</tr>
</tbody>
</table>

1. Year with 27% increase in tutor hours.
Other Colleges’ Approaches....

- Hollins University – Big Q course not necessarily focused on data analysis; incorporates at least two quantitative projects
- Johnson State College – Offers “quantitatively enriched courses”
- Carleton College – Focus on QR in writing – in supporting arguments with quantitative evidence when appropriate
How Might You Approach QR at Holy Cross?

- Must fit your school’s culture, climate, curricular needs....
- What’s your vision? What are your goals?
- How would you start up? Further pilot programs beyond using QLRA?
- What sources of funding might you pursue?
- How will you monitor the program?
- What are your measures of success?
Considerations for QR and Math at Holy Cross

- Give a QR Assessment to all incoming students during Orientation as a benchmark for preparedness
- Enhance F.Y. Seminars with more quantitative components
- Reconsider what programs and courses count for the Math Requirement (3-week summer bridge course; BioStats course)
You have support from QR colleagues!