Endogenous Separation, Wage Rigidity and the Dynamics of Unemployment*

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Abstract

Previous attempts to generate sufficient unemployment volatility in the Mortensen-Pissarides model rely on either endogenous separation or wage rigidity. In this paper I simulate a version of the Mortensen-Pissarides (MP) model with both wage rigidity and endogenous separation. I find the model generates sufficient volatility in unemployment, the separation rate and the finding rate, 75% of the observed volatility in vacancies, and 70% of the Beveridge curve (the negative correlation between unemployment and vacancies). The model matches the volatility of the average wage and does not generate counterfactually low responses of the wage of new hires to productivity and unemployment. I then simulate the model while restricting the separation rate to be constant and show that the model predicts only 70% of the variance of unemployment though the model is more consistent with the volatility of vacancies and the Beveridge curve. I conclude that finding rate fluctuations explain 70% of unemployment fluctuations halfway in between the most prominent estimates in the literature.

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1 Introduction

The Mortensen & Pissarides (1994) (MP) search and matching model is the dominant paradigm for studying unemployment fluctuations. Shimer (2005a) argues that the model is unable to explain the volatility of unemployment, job-finding and vacancies. One strand of the literature, (e.g. Hall (2005a), Gertler & Trigari (2009)) attempts to rectify the model's shortcomings by allowing for wage rigidity. However this work assumes a constant separation rate. While earlier work (e.g. Shimer (2005b), Hall (2005b)) has argued that the separation rate is relatively acyclical and contributes little to unemployment fluctuation, much recent work has demonstrated a role for variation in the separation rate in explaining unemployment fluctuations. For example, Elsby *et al.* (2009) which concludes, "A complete understanding of cyclical unemployment requires an explanation of countercyclical inflow rates." Models with a constant separation rate, then, can only give an incomplete explanation for unemployment fluctuations.

Noting that the separation rate appears to be endogenous, another strand of the literature (e.g. Ramey (2008) and Menzio & Shi (2009)) focuses on modeling endogenous separation. However these models suffer from the Shimer puzzle (Shimer (2005a)) i.e. they generate too little variance in the job finding rate.

Both strands of the literature are important contributions to our understanding of unemployment fluctuations. However, since both strands underestimate the importance of one channel in generating unemployment fluctuations (either job-finding or job-separation) neither can fully explain the volatility in unemployment fluctuations. My paper bridges the gap between these two approaches. I examine if wage rigidity and endogenous separation together can give a complete explanation of unemployment volatility. I solve a version of the MP model with endogenous separation and wage rigidity. The model is able to explain the volatility of unemployment, the separation rate, the finding rate and the average wage. The model does not predict a counterfactually low response of the wages of new hires to unemployment or productivity. The model explains 74% the volatility of vacancies and 70% of

the Bevridge Curve (the strong negative correlation between unemployment and vacancies).

I also show that restricting the separation rate to be constant allows the model to better explain the Bevridge curve and the volatility of vacancies, but the model can not explain the observed volatility of unemployment. The constant separtion rate model explains only 70% of the observed volatility of unemployment. This estimate is halfway between the two most prominent estimates of the contribution of the finding rate to unemployment fluctuations.

The rest of the paper proceeds as follows. Section two describes the baseline model with endogenous separation and wage rigidity. Section three explains the model solution, calibration and gives the results from simulating the model. In section four, I demonstrate that once the separation rate is restricted to be constant, the model predicts too little variance in unemployment. In Section 5 I compare the model to other attempts to quantify the importance of the separation rate in explaining unemployment fluctuations. Section six concludes.

2 Model

2.1 Theoretical Model

2.1.1 Informal Description

In this section I describe a version of the Mortensen & Pissarides (1994) model with wage rigidity. The model is a discrete time version of the Mortensen-Pissarides model. The model has large, persistent idiosyncratic productivity shocks to allow for endogenous separation. Some matches will be hit with a negative enough productivity shock that the value of unemployment exceeds the value of production. These matches separate and the worker becomes unemployed.

My main departure from the standard MP model is the inclusion of wage rigidity. Shimer (2005a) demonstrates that the Mortensen-Pissarides model does not generate sufficient un-

employment volatility when workers' outside options are low. Finding this also to be the case for my model as well, I add wage stickiness via a wage norm, as in Hall (2005a), to increase the model's ability to generate unemployment volatility.

Because wages are rigid, and there are large idiosyncratic productivity shocks, wages may be, at times outside the bargaining set of the worker and firm. If this is the case, I assume that the outside option binds and that the wage adjusts to avoid an inefficient separation. Therefore, if the match receives a shock such that given the wage is too high and the firm will want to sever the match, the wage adjusts so the firm's share of the surplus is zero. This makes the firm indifferent between keeping and firing the worker. Similarly if the wage is too low such that the worker would want to quit the match, I assume that the wage rises so that the worker gets a zero share of the surplus. I now proceed to a formal description of the model.

2.1.2 Match Productivity

At the beginning of the period there is a mass of worker-firm matches. Workers maximize expected discounted lifetime income. Firms maximize expected discounted profits. A fraction ρ^x of matches exogenously separates into unemployment[1]. Remaining firms produce according to the following production function $x_{i,t}y_t$ where $x_{i,t} = x_{i,t-1}$ with probability $1-\phi$ and with probability ϕ, $x_{i,t}$ is drawn from the discrete distribution $H(x)$ with maximum value x^h. y_t represents aggregate productivity, which follows the $AR(1)$ process $\ln y_t = \rho^z \ln y_{t-1} + \varepsilon_t$ where ε_t is an i.i.d normal random variable with mean 0 and standard deviation σ^ε.

[1] Exogenous separation can be thought of as needing to leave a job for personal reasons or as receiving a permanent shock that destroys the value of the job.

2.1.3 Match Surplus and Separation

After observing the idiosyncratic and aggregate levels of productivity, the pairs calculate the expected surplus of remaining in the match:

$$S_t(x_{i,t}) = x_{i,t}y_t \ + G_t(x_{i,t}) - (U_t + b) \tag{1}$$

$G_t(x_{i,t})$ represents the expected future discounted value to the firm and the worker if they remain in a match with idiosyncratic productivity level $x_{i,t}$, U_t represents the future benefits that will accrue to the worker if she is unemployed this period, and b represents the flow value of being unemployed. Note that the surplus is the value of the match in excess of the worker's outside option, the value of unemployment. The firm's outside option is normalized to zero.

Since $S_t(x_{i,t})$ is increasing in $x_{i,t}$ there is a threshold value of idiosyncratic productivity (x_t^*) below which the surplus is zero and the match is terminated:

$$0 = \ x_t^* y_t \ + G_t(x_t^*) - (U_t + b) \tag{2}$$

2.1.4 Wage Setting

The standard MP model assumes that wages are perfectly flexible and adjust so that the firm gets a share π of the surplus. In this model I take the approach of Hall (2005a) and assume that wages are not perfectly flexible. Instead, wages are a weighted average between the wage that would give the firm a share π of the surplus and a wage norm. This assumption allows the firm's share of the surplus to vary over time. After a negative aggregate productivity shock, the wage does not adjust fully downward and the firm gets a share smaller than π of the surplus. This reduces their incentive to recruit and lowers the job-finding rate. This mechanism can generate additional volatility in job-finding. Wages

then are given by:

$$w_t(x_i) = \lambda \overline{w_i} + (1-\lambda)((1-\pi)x_i y_t + \pi b + F_t(x_i) - \pi(G_t(x_i) - U_t)) \tag{3}$$

where $F_t(x_i)$ is the future expected discounted payments that accrue to the firm in a match with idiosyncratic productivity x_i. $(1-\pi)x_i y_t + \pi b + F_t(x_i) - \pi(G_t(x_i) - U_t)$ is the wage that, when paid, would give the firm a share π of the total surplus. $\overline{w_i}$ represents a wage norm that will be defined shortly. λ is a measure of wage stickiness. The closer λ is to one, the more rigid are wages.

Since wages are rigid, it is possible that the idiosyncratic productivity level is low enough that the firm would want to fire the worker when there is positive value in the match (i.e. $x_i > x_t^*$). The firm would want to fire the worker if:

$$x_i y_t - w_t(x_i) + F_t(x_i) < 0 \tag{4}$$

Here the value of the match to the firm $x_i y_t + F_t(x_i)$ is less than the wage it must pay. If the firm would want to fire the worker at the wage given by the wage rule, I assume that the wage adjusts so that the firm's share of the surplus is equal to zero, i.e. the firm is indifferent between keeping or firing the worker. In this case the wage is given by:

$$w_t(x_i) = x_{i,t} y_t + F_t(x_{i,t}) \tag{5}$$

It is also possible that the wage is too low and the worker would want to exit the match. This event would occur if

$$w_t(x_i) + G_t(x_i) - F_t(x_i) - (U_t + b) < 0 \tag{6}$$

Here the value of the worker's outside option $(U_t + b)$ is greater than the value of the job to

the worker: the wage $w_t(x_i)$ plus the expected future discounted payments that accrues to the worker $G_t(x_i) - F_t(x_i)$ (the joint firm and worker value minus the value that accrues to the firm). In this case I assume that the wage adjusts so that the worker is indifferent between quiting the job and staying. Then wages are given by:

$$w_t(x_i) = (U_t + b) - G_t(x_i) + F_t(x_i) \tag{7}$$

I assume that the wage norm is the wage that gives the firm a share π of the surplus in steady state ($where\ y_t = 1$). Therfore

$$\overline{w_i} = (1-\pi)x_i + \pi b + F^{ss}(x_i) - \pi(G^{ss}(x_i) - U^{ss}) \tag{8}$$

2.1.5 Continuation Value Functions

To solve the model it is necessary to calculate the continuation value functions. The expected future payments of the match to the firm satisfy:

$$F_t(x_i) = \beta(1-\rho^x)E_t \left[\begin{array}{c} (1-\phi)[x_i y_{t+1} - w_{t+1}(x_i) + F_{t+1}(x_i)] * 1(x_i > x^*_{t+1}) \\ +\phi \int_{x^*_{t+1}}^{x^h} [\widetilde{x}_i y_{t+1} - w_{t+1}(\widetilde{x}_i) + F_{t+1}(\widetilde{x}_i)] dH(\widetilde{x}_i) \end{array} \right] \tag{9}$$

The firm discounts future payments at a rate β, and the match remains with probability $(1-\rho^x)$. With probability $(1-\phi)$, $x_{i,t+1} = x_i$ and the match remains if $x_{i,t+1} > x^*_{t+1}$ (hence the need for the indicator function $1(\cdot)$). With probability ϕ the match receives a new value for x_i. If the idiosyncratic productivity shock is above x^*_{t+1}, the match produces. The firm collects $\widetilde{x}_i y_{t+1}$, pays the worker $w_{t+1}(\widetilde{x}_i)$, and the match has continuation value $F_{t+1}(\widetilde{x}_i)$ to the firm.

The total expected payments from remaining in the match today which accrue to either

the firm or the worker satisfy:

$$G_t(x_i) = \beta E_t \left[(1-\rho^x) \left(\begin{array}{c} (1-\phi)\max(x_i y_{t+1} \ + G_{t+1}(x_i) - (U_{t+1}+b), 0) \\ +\phi \int_{x^*_{t+1}}^{x^h} [\widetilde{x}_i y_{t+1} \ + G_{t+1}(\widetilde{x}_i) - (U_{t+1}+b)] dH(\widetilde{x}_i) \end{array} \right) + U_{t+1} + b \right] \tag{10}$$

In the event that the match does not separate exogenously, with probability $(1-\phi)$, $x_{i,t+1} = x_i$ and the surplus of the match is given by $\max(x_i y_{t+1} \ + G_{t+1}(x_i) - (U_{t+1}+b), 0)$. With probability ϕ the match receives a new value for x_i, and in the case $\widetilde{x}_i > x^*_{i,t+1}$ the match has surplus value $x_i y_{t+1} \ + G_{t+1}(x_i) - (U_{t+1}+b)$. In any event, the worker is guaranteed her outside option $U_{t+1} + b$.

Finally, the value of unemployment is:

$$U_t = \beta E_t \left[\frac{m_t}{u_t}(1-\rho^x) \left(\begin{array}{c} (1-\phi)[w_{t+1}(x^h) + G_{t+1}(x^h) - F_{t+1}(x^h) - (U_{t+1}+b)] \\ +\phi \int_{x^*_{t+1}}^{x^h} [w_{t+1}(x_i) + G_{t+1}(x_i) - F_{t+1}(x_i) - (U_{t+1}+b)] dH(x_i) \end{array} \right) + U_{t+1} + b \right] \tag{11}$$

The worker finds a job with probability $\frac{m_t}{u_t}$ (the number of matches per unemployed worker) and with probability $(1-\rho^x)$ the match does not separate exogenously. I assume that all matches begin at the highest idiosyncratic productivity level, though the idiosyncratic productivity level can change before production.[2] Therefore, with probability $(1-\phi)$, $x_i = x^h$ and the worker gets surplus $w_{t+1}(x^h) + G_{t+1}(x^h) - F_{t+1}(x^h) - (U_{t+1}+b)$. With probability ϕ the match receives a new value for x_i and in the case $x_{i,t+1} > x^*_{i,t+1}$ the worker surplus equals $w_{t+1}(\widetilde{x}_i) + G_{t+1}(\widetilde{x}_i) - F_{t+1}(\widetilde{x}_i) - (U_{t+1}+b)$. In all cases, the worker receives her outside option $U_{t+1} + b$.

[2] I find this assumption convenient computationally. However, it is not necessary for the results.

2.1.6 Employment Dynamics

Workers separate from jobs into unemployment and I calculate flows from employment to unemployment as:

$$EU_{t+1} = \rho^x n_t + (1-\rho^x)[\phi H(x^*_{t+1})n_t + (1-\phi)\sum_{x_i < x^*_{t+1}} n_t(x_i)] \tag{12}$$

here n_t is the total number of matches which produce at time t and $n_t(x_i)$ is the total number of matches that produce at production level x_i. A fraction ρ^x of all matches separate exogenously into unemployment each period. In addition, a fraction $\phi H(x^*_{t+1})$ will have idiosyncratic productivity next period below the separation threshold as will all those for which x_i remains less than x^*_{t+1}.

The separation rate is given by

$$s_{t+1} = \frac{EU_{t+1}}{n_t} \tag{13}$$

I assume that new matches begin at the highest productivity level (x^h) but are subject to the idiosyncratic shock so that next period's employment stocks are given by:

$$n_{t+1}(x_i) = 0 \qquad \text{if } x_i <= x^*_{t+1} \tag{14}$$

$$n_{t+1}(x_i) = (1-\rho^x)\phi h(x_i)(n_t + m_t) + (1-\rho^x)(1-\phi)n_t(x_i) \qquad \text{if } x^*_{t+1} < x_i < x^h \tag{15}$$

$$n_{t+1}(x^h) = (1-\rho^x)\phi h(x^h)(n_t + m_t) + (1-\rho^x)(1-\phi)(n_t(x^h) + m_t) \tag{16}$$

where $h(x_i)$ is the probability that the draw from the distribution $H(x) = x_i$. All matches with productivity less than or equal to x^*_{t+1} are destroyed. For matches with productivity between $x^*_{t+1} < x_i < x^h$, a fraction $(1-\rho^x)(1-\phi)$ remain at the same productivity level and they are joined by $(1-\rho^x)\phi h(x_i)(n_t + m_t)$ who receive a new idiosyncratic productivity level equal to x_i. For matches with productivity $x_i = x^h$ the calculation is the same, but we

take into account that the new matches also begin at productivity level x^h.

Finally, unemployment evolves according to:

$$u_{t+1} = u_t + EU_{t+1} - (1-\rho^x)[(1-\phi)m_t + \phi(1-H(x^*_{t+1}))m_t] \tag{17}$$

and the size of the labor force is normalized to 1 so that $n_t + u_t = 1$. Here inflow into unemployment equals EU_{t+1} and outflow from unemployment equals the number of matches times the fraction that do not separate back into unemployment.

The next two equations determine the equilibrium number of matches and vacancies. Firms post vacancies up to the point where the marginal benefit of doing so equals the marginal cost c:

$$c = \frac{m_t}{v_t} F_t(x^h) \tag{18}$$

$F_t(x^h)$ is the value today of filling a vacancy and $\frac{m_t}{v_t}$ is the likelihood that the vacancy is filled.

The following Cobb-Douglas function determines the number of matches:

$$m_t = A u_t^{\alpha} v_t^{1-\alpha} \tag{19}$$

This matching function exhibits constant returns to scale and m is increasing in u and v. Moreover, $\frac{m}{v}$ and the finding rate, $\frac{m}{u}$, are decreasing in v and u respectively. This type of random matching function is meant to model frictions in the labor market. Unemployed workers cannot immediately find a job, but do so randomly with a probability less than one.

2.1.7 Timeline

To summarize:

1. At time t there is a vector $n_t(x_i)$ of individuals employed and producing in each job and a number of unemployed u_t.

2. Firms post vacancies and the number of matches, m_t, is determined by the free entry condition (18) and the matching function (19).

3. The the next level of aggregate productivity y_{t+1} is drawn and then a fraction ϕ of all matches (new and old) receive a new value of x_i drawn from the distribution $H(x)$.

4. A fraction of all matches ρ^x separate into unemployment.

5. All matches for which $x_i < x^*_{t+1}$ separate into unemployment.

6. Remaining matches produce and firms post vacancies to determine $m_{t+1.}$

2.1.8 Equilibrium

The equilibrium of the model is a set of function $F(x_i, y_i)$, $G(x_i, y_i)$, $U(y_i)$ satisfying the continuation value functions equations (9), (10) and (11). A vector $x(y_i)$ solving the separation threshold equation (2), a wage setting function, $w(x_i, y_i)$ satisfying the equations of the wage setting section (3), (5) and (7), and a vector of market tightness $\theta(y_i) \equiv \frac{v_t}{u_t}$ satisfying the free entry condition (18) and the matching function (19).

2.2 Empirical Motivation for Wage Rigidity

Real-wage rigidity is an important feature of the model in this paper. As pointed out by Shimer (2005a), the standard Mortensen-Pissarides model does not generate sufficient volatility in unemployment and vacancies. Hall (2005a) notes that the model's amplification mechanisms are greatly improved by adding real wage rigidity. This is true even if the wage is allowed to adjust to avoid inefficient separations. Substantial real-wage rigidity moves the model towards paying the worker a wage that varies less with the state of aggregate productivity. As a result, the firm keeps gains from aggregate-productivity increases and absorbs losses from aggregate-productivity decreases. This mechanism makes the firm's recruiting incentives more procyclical, generating more variance in unemployment and vacancies through the finding rate.

Beyond the empirical necessity, additional research points to the importance of real wage

rigidity. Hall (2005a) argues that there is a social consensus as to what the fair wage is and that a sense of a fair wage may affect wage setting. Akerlof *et al.* (1996) and Bewley (1999) support this view as well. Falk *et al.* (2006) introduce minimum wages in experimental settings. They find introducing minimum wages raises reservations wages. Even after removing the minimum wage, the reservation wages remain higher than before. They argue that the minimum wage shapes what subjects consider a fair wage.

While the average wage's relative acyclicality is well known, the cyclicality of new hires' wages is currently an active research area. As noted by Pissarides (2009) and Haefke *et al.* (2008), in a sample of those who have begun work recently, wages are quite sensitive to changes in aggregate productivity. For example, Haefke *et al.* (2008) find that a 1% increase in productivity leads to a 0.79% increase in the wage of new hires. This evidence would seem to cast doubt on the ability of wage rigidity to explain fluctuations in job-finding.

However, as Gertler & Trigari (2009) argue, these studies fail to control for changes in the type of job at which workers work. For example, if in recessions workers transition more from well paying jobs (e.g. manufacturing) to poorer paying jobs (e.g. retail), wages of new hires will be very sensitive to aggregate productivity. After controlling for job-specific characteristics, they find that wages of new hires are no more sensitive to the aggregate state of the economy than those of current employees.

Recent evidence from Portugal attempts to address the argument of Gertler and Triagari. Carneiro *et al.* (2009) estimate a wage equation with firm fixed effects and find that the cyclicality of the wage for new hires is twice the cyclicaltiy of the wage for those continuing in the same job. However, Martins *et al.* (2010), point out that if cyclical upgrading of job-type or match-specific quality, as opposed to firm quality, is important. Firm fixed effects will not capture this effect, overestimating the cyclicality of the wage of new hires.

Noting this critique, among others, they use the same data and find much less cyclicality in the wage of new hires after controlling for job-specific characteristics in a different manner. They construct a time series of entry jobs and find little difference between the cyclicality

volatility of these job's wages and overall wages. (Table 2 row 11 vs. Table 3 rows 1 and 2). However, they do confirm the finding that micro level estimates of wage cyclicality are higher than aggregate measures of wage cyclicality. They find a 1 point increase in the unemployment rate corresponds to a 1.79% decrease in the entry level job wage.

For my model I find that a 1 percentage point increase in the unemployment rate corresponds to a 1.1% decrease in the new hire wage, within the confidence interval of the Martins *et al.* (2010) estimate.[3] Similarly, I find that the wage of new hires increases by 0.7% when aggregate productivity increases by 1%, only slightly below the estimates of Haefke *et al.* (2008). Hence, the wage stickiness in my model does not imply a counterfactually acyclical wage for new hires.

Another issue of concern may be that the model's adjustment of wages to avoid an inefficient separation may result in lower volatility for the wages of new hires versus the average wage. However, I find that the wages of new hires is 94% as volatile as the average wage lessening this concern.

3 Model Solution, Estimation, and Results

3.1 Model Solution

To solve the model I discretize aggregate productivity $\ln y$ using the method of Tauchen & Hussey (1991). This method gives a grid of possible values for $\ln y$, $y = \{y_1 ... y_n\}$ and a transition matrix $\Pi_{ij} = \text{Prob}(y_{t+1} = y_i |\ y_t = y_j)$. I set $n = 21$. I then guess initial functions $F(x_i, y)$, $G(x_i, y)$, and $U(y)$ and iterate on equations (9,10 and 11) until the functions converge.[4]

To approximate the distribution for the idiosyncratic shocks I follow Ramey (2008)

[3] This value is calculated as the regression coefficient β in $\ln w_t^h = \alpha + \beta u_t + \varepsilon_t$. Where w^h is the new hire wage. $\beta = \frac{cov(u_t, w_t^h)}{var(u_t)}$.

[4] As common in the literature, I look for an equilibrium where the value function depends only on idiosyncratic and aggregate productivity and not the employment distribution. Existence follows by construction however I am unable to demonstrate uniqueness.

Table 1: Endogenous Separation Model Parameters

Parameter	Symbol	Value	Source
Discount factor	β	$0.99^{1/12}$	Standard
Productivity autoregressive parameter	ρ^z	$0.96^{1/12}$	Standard
Aggregate productivity shock stdev.	σ^ε	0.002605	Stdev. of productivity
Exogenous separation rate	ρ^x	0.00542	Silva and Toledo (2009)
Flow value of unemployment	b	0.75	Costain and Reiter (2003)
Sticky wage weight	λ	0.924	Stdev. of finding rate
Matching Function Elasticity	α	0.5246	Hoisos (1990)
Probability of new idiosyncratic shock	φ	0.168	Stdev. of separation rate
Vacancy posting cost	c	0.3741	Match: mean finding rate (Shimer (2005)), mean firm matching rate (Davis et. al (2010)), mean separation rate (Hall (2005b)), recruiting cost (Hall and Milgrom (2008)), mean productivity equal to 1.
Matching function efficiency	A	0.1621	
Firm's bargaining share	π	0.4754	
Idiosyncratic standard deviation	σ^z	0.2689	
Highest idiosyncratic productivity	x^h	1.2077	

This table describes the parameters from the main model in the paper and explains how the variables are calibrated. It also gives the values of the calibrated parameters.

and let $x_k = \{x_1, ..., x_K\}$ with $x_K = x^h$. I let $x_1 = 1/K$, $x_k - x_{k-1} = \frac{x^h}{K}$ and set $\text{Prob}(x_i = x_j) = \frac{x^h \log norm(x_j, 0, \sigma^z)}{K}$ for $j = 1, ..., K-1$ and $\text{Prob}(x_i = x_K) = 1 - \sum_{i=1}^{K-1} \text{Prob}(x_i = x_j)$.[5] $\log norm(x_j, 0, \sigma_z)$ denotes the lognormal p.d.f. evaluated at x_j with mean zero and standard deviation σ^z. I set K=200.

3.2 Calibration

Table one contains the parameters. The model is calibrated at a weekly frequency and I assume four weeks per month. I set the discount factor $\beta = 0.99^{\frac{1}{12}}$ and the productivity

[5] Ramey (2008) uses $\frac{\log norm(x_j, 0, \sigma^z)}{K}$ instead of $\frac{x^h \log norm(x_j, 0, \sigma^z)}{K}$ though I find the difference to be unimportant.

autocorrelation parameter $\rho^z = 0.96^{\frac{1}{12}}$.[6] Following Silva & Toledo (2009) I set the exogenous separation probability to $\rho^x = 0.00542$ which corresponds to their quarterly probability of 0.065. I then set the standard deviation of the productivity shocks $\sigma^\varepsilon = 0.002605$ in order to match the standard deviation of HP-filtered aggregate labor productivity.[7] As suggested by Costain & Reiter (2003) I set $b = 0.75$ to be consistent with the response of unemployment to changes in the unemployment insurance replacement ratio.[8] I set $\lambda = 0.924$ to match the standard deviation of the finding rate and $\phi = 0.168$ to match the standard deviation of the separation rate. I set $\alpha = 1 - \pi$ (the worker's bargainign share) the so called Hosios (1990) condition. Finally $c = 0.3741$, $A = 0.1621$, $x^h = 1.2077$, $\sigma^z = 0.2689$, and $\pi = 0.4754$. To calibrate these parameters I match the following moments in steady state. I normalize mean productivity across jobs $(\frac{\sum_i x_i * n(x_i)}{\sum_i n(x_i)}) = 1$. I set the steady state finding rate to 0.12 which implies a monthly finding rate of 0.40 consistent with the evidence in Shimer (2005a). I set the overall weekly separation rate to 0.0082 which translates to a monthly separation rate of 0.0323 consistent with the estimate in Hall (2005b). I set the weekly vacancy filling probability $(\frac{m}{v})$ equal to 0.226 which corresponds to a daily vacancy filing rate of 5% as in Davis *et al.* (2009). And I set the steady state job filling cost $\frac{c}{m/v} = 14\%$ the quarterly wage as in Hall & Milgrom (2008).

[6] At a quaterly frequency it is common to assume an AR(1) parameter of 0.96, see for example Silva & Toledo (2009). This motivates my choice of $0.96^{1/12}$. I find that in my model the autocorrlation of quaterly productivity is 0.74 vs. 0.79 in the data. However, since I HP-filter the quaterly data with a smoothing parameter of 1600, I can not match the autocorrelation of 0.79 in the data even with higher levels of weekly persistance. This is because the quaterly averaging and the HP-filtering results in a persistance below the data even as $\rho^z \to 1$. As an additional robustness check, I find that adpoting the weekly value of 0.99 from Ramey (2008) did not change the results.

[7] Aggregate productivity is given by $\frac{y_t \sum_i x_i * n_t(x_i)}{\sum_i n_t(x_i)}$ where $n_t(x_i)$ is employment in job i at time t.

[8] As in Costain and Reiter I find the steady state semi-elasticity of unemployment $\frac{\partial \ln u_t}{\partial b} = 2$ in my model when $b = 0.75$.

3.3 Model Simulation

To simulate the model I run 1,000 trials beginning at the steady state level of productivity and employment. I simulate 3,000 weeks and I keep only the last 188*3*4= 2,256 weeks of data to match the length of my data sample. I average the data at the quarterly level and HP-filter the log variables with a smoothing parameter of 1600.[9] I then report the median values across the 1,000 trials.

3.4 Data

The data are as in Shimer (2005a) except data begin in 1964 and end in 2010.[10] The measure of unemployment is the seasonally adjusted civilian unemployment rate from the Bureau of Labor Statistics (BLS). The series on vacancies is the Conference Board's Help Wanted advertisements series.[11] Productivity is measured as real output per person in the non-farm business sector and come from the BLS Major Sector Productivity and Costs Program. The measure of wages is the BLS series on average hourly earning of production and non-supervisory employes in the private sector deflated with the consumer price index (CPI). This is the wage data used in Gertler & Trigari (2009). I follow the methodology of Shimer (2005b) (section 1) to construct series for the monthly job-finding probabilities and job-separation probabilities from the BLS's Current Population Survey (CPS) through 2010.[12] All data except for the productivity data are quarterly averages of monthly series. All data are transformed in logarithms and HP filtered with a smoothing parameter 1600.

Table 2: Endogenous Separation Model Results

	σ(u)	σ(v)	σ(f)	σ(s)	σ(w)	σ(p)
Data	0.118	0.136	0.082	0.052	0.011	0.013
Model	0.13	0.1	0.082	0.052	0.011	0.013
	ρ(u,v)	ρ(u,f)	ρ(u,s)	ρ(v,f)	ρ(v,s)	ρ(f,s)
Data	-0.91	-0.94	0.41	0.86	-0.52	-0.21
Model	-0.64	-0.92	0.87	0.89	-0.78	-0.9
	ρ(u,p)	ρ(f,p)	ρ(s,p)	ρ(v,p)	ρ(w,p)	
Data	-0.24	0.18	-0.57	0.44	0.55	
Model	-91	0.99	-0.92	0.88	0.99	

This table contains the results of simulating the main model in the paper. Simulations of length 3000 are run. And I keep the last 3*4*188 observations to match the 188 quarters of data. This table reports the median across all simulations.

3.5 Endogenous Separation Model Results

Table two contains the main moments in the data. The results are comparable to Shimer (2005a) though I uses a lower, more conventional smoothing parameter (1600 versus 10^5) so the standard deviations are smaller.[13] The standard deviation of unemployment is about 12% and it is 13.6% for vacancies. The finding rate with a standard deviation of 8% is more volatile than the separation rate which has a standard deviation of 5%. The standard deviation for wages and productivity is about 1%. There is a strong negative correlation between unemployment and vacancies (the Bevridge curve) and unemployment and the finding rate. The correlation between the separation rate and unemployment is lower as is the correlation between the separation rate and the finding rate. The correlations with productivity have all the expected signs. However, in the last few years of the sample the correlation between the finding rate, unemployment and productivity has changed signs

[9] To compare the model to the data I convert the weekly model probabilities to monthly probabilities. Therfore, I set $f^m = 1 - (1 - f^w)^4 and\ s^m = 1 - (1 - s^w)^4$. Simlar results were obtianed comparing weekly model transition rates to monthly rates calcualtes as in Shimer (2005b).

[10] Wage data begin only in 1964.

[11] I thank Ken Goldstein at the Conference Board for providing me the data through 2010.

[12] This method uses data on unemployment and short term unemployment to infer the job-finding and job-separtion probabilities correcting for time aggregation bias. The method also requires data from the incoming rotation groups to correct for changes in the CPS survey design beginning in 1994.

[13] Using a smoothing parameters of 10^5 did not change the evaluation of the model's fit to the data or the main conclusions of the paper.

resulting in a low average correlation between these variables.

Table two also contains the results from simulating the model. Recall that the parameters of the model are chosen so that the model exactly matches the observed standard deviation of the job-finding rate, the separation rate and productivity.

For the moments I do not target with the calibration the model also performs well. The model predicts a standard deviation of unemployment equal to 0.13 close to the 0.118 in the data. The model explains 74% the volatility of vacancies. The model is able to do this while also exactly matching the volatility of wages. This is not by construction. Wage stickiness is calibrated to match the volatility of the finding rate; that the model then gives realistic volatility for the wage rate is remarkable. (Recall, as discussed in section 2.2 the model also gives reasonable predictions for the cyclicality of the wage of new hires.)

The model predicts all the correct signs for the correlation coefficients. It comes very close to matching the correlation of unemployment and the finding rate (-0.92 vs. -0.94 in the data), the correlation of vacancies and the finding rate (0.89 versus 0.86 in the data). The model overestimated the correlation between unemployment and the separation rate (0.87 versus 0.41 in the data), the correlation between vacancies and the separation rate (-0.78 versus -0.52 in the data), and the correlation between the separation rate and the finding rate (-0.9 versus -0.64 in the data). Finally, the model can not fully match the Bevridge curve. It predicts the correlation between unemployment and vacancies should be -0.64 versus -0.91. Many models with endogenous separation predict a positive correlation between unemployment and vacancies as the increase in separations causes both an increase in unemployment and vacancies. That is not the case here because the presence of wage rigidity results in a volatile finding rate that counteracts this effect.

For the correlations with productivity, the model replicates the correct signs. However, it implies correlations with productvitiy that are too high because productivity is the only shock in the model.

It appears then that the standard MP model does a good job capturing the main moments

in the data once it is allowed to have reasonable levels of wage rigidity and an endogenous separation rate.

4 Constant Separation Rate Model

This section examines the model without an endogenous separation rate. Doing so demonstrates the need for endogenous separation to explain the volatility of unemployment and also gives an answer to the question of what percent of unemployment fluctuations come from changes in the separation rate. In the next section I compare my answer with other answers provided in the literature.

To study these questions I remove the idiosyncratic productivity shocks. Now for all matches $x_{i,t} = 1$. Again, I allow wages to be rigid. Now wages then given by[14]:

$$w_t = \lambda \overline{w} + (1-\lambda)((1-\pi)y_t + \pi b + F_t - \pi(G_t - U_t)) \tag{20}$$

Again the wage norm is the wage in steady state (where productivity y is normalized to 1) that would give the firm a share π of the surplus.

$$\overline{w} = (1-\pi) + \pi b + F^{ss} - \pi(G^{ss} - U^{ss}) \tag{21}$$

The continuation values are given by:

$$F_t = \beta(1-\rho^x)E_t[y_{t+1} - w_{t+1} + F_{t+1}] \tag{22}$$

$$G_t = \beta E_t\left[(1-\rho^x)\left(y_{t+1} + G_{t+1} - U_{t+1} + b\right) + U_{t+1} + b\right] \tag{23}$$

$$U_t = \beta E_t\left[\frac{m_t}{u_t}(1-\rho^x)\left(w_{t+1} + G_{t+1} - F_{t+1} - (U_{t+1} + b)\right) + U_{t+1} + b\right] \tag{24}$$

[14] I find without the idiosyncratic productivity shocks the wage rule results in wages always in the bargaining set. Therefore, there is no need to adjust the wage in this case.

Employment and unemployment evolve according to:

$$n_{t+1} = (1-\rho^x)(n_t + m_t) \tag{25}$$

$$u_{t+1} = u_t + \rho^x n_t - (1-\rho^x) m_t \tag{26}$$

And the free entry condition becomes

$$c = \frac{m_t}{v_t} F_t \tag{27}$$

Table 3: Parameters in the Constant Separation Rate Model

Parameter	Symbol	Value	Source
Discount factor	β	$0.99^{1/12}$	Standard
Productivity autoregressive parameter	ρ^z	$0.96^{1/12}$	Standard
Aggregate productivity shock stdev.	σ^ε	0.00239	Stdev. of productivity
Exogenous separation rate	ρ^x	0.00817	Hall (2005b)
Flow value of unemployment	b	0.75	Costain and Reiter (2003)
Sticky wage weight	λ	0.93125	Stdev. of finding rate
Matching Function Elasticity	α	0.5238	Hoisos (1990)
Vacancy posting cost	c	0.374	Match: mean finding rate (Shimer (2005)), mean firm matching rate (Davis et. al (2010)), recruiting cost (Hall and Milgrom (2008)).
Matching function parameter	A	0.1622	
Firm's bargaining share	π	0.4762	

This table describes the parameters from the main model in the paper and explains how the variables are calibrated. It also gives the values of the calibrated parameters.

Calibration of the constant separation rate model is very similar to calibration of the endogenous separation rate model and summarized in Table 3. However I no longer need to calibrate the parameters associated with the idiosyncratic productivity process: ϕ (the probability of a new shock), σ^z (the standard deviation of the idiosyncratic productivity process) and x^h (the highest level of productivity). Also, I set the exogenous separation rate

ρ^x to 0.0082 consistent with an overall monthly separation rate of 0.0323.

4.1 Constant Separation Results

Table 4: Constant Separation Rate Model Results

	σ(u)	σ(v)	σ(f)	σ(s)	σ(w)	σ(p)
Data	0.118	0.136	0.082	0.052	0.011	0.013
Model	0.085	0.134	0.082	---	0.01	0.013
	ρ(u,v)	ρ(u,f)	ρ(u,s)	ρ(v,f)	ρ(v,s)	ρ(f,s)
Data	-0.91	-0.93	0.41	0.86	-0.52	-0.21
Model	-0.79	-0.91	---	0.96	---	---
	ρ(u,p)	ρ(f,p)	ρ(s,p)	ρ(v,p)	ρ(w,p)	
Data	-0.24	0.18	-0.57	0.44	0.55	
Model	-0.91	0.98	---	0.95	0.99	

This table contains the results of simulating the constant separation rate model in the paper. Simulations of length 3000 are run. And I keep the last 3*4*188 observations to match the 188 quarters of data. This table report the median across all simulations

The model with a constant separation rate is no longer able to match the volatility of unemployment. It predicts a volatility of 0.085 versus 0.118 in the data. However, it almost completely explains the volatility of vacancies (0.134 versus 0.136 in the data). The constant separtion rate model creates more volatility of vacancies than the endogenous separation rate model. In the endogenous separation rate model, additional separations during recessions leads to more demand for vacancies which mutes the fall in vacancies that normally occurs in recessions. This mechanism is not present in the constant separation rate model. The model also improves on the fit of the Bevridge curve (-0.79 versus -0.91 in the data), and matches almost exactly the correlation between the finding rate and unemployment and vacancies. The correlations with productivity are very similar to the model with endogenous separation.

This section demonstrates then that: 1. endogenous separation rates are necessary to match the volatility of unemployment however 2. endogenous separation makes it somewhat harder for the model to explain the volatility of vacancies and the negative correlation between unemployment and vacancies.

5 Comparision to Other Decompositions

The model then gives an answer to what percent of unemployment fluctuations are due to variations in the separation rate. Two other answers to this question have been provided by using data on unemployment fluctuations, separation rates and finding rates. I describe each of these decompositions and compare those conclusions to the conclusion of my model.

Shimer (2007) begins with quarterly average of monthly observations on the one month ahead unemployment rate, separation rate and job-finding rate. He then notes that the steady state unemployment rate is given by $u^{ss} = s/(s+f)$ where s is the separation rate and f is the job-finding rate. He creates two hypothetical measures of the unemployment rate:

$$u_t^f = \frac{\overline{s}}{\overline{s} + f_t} \tag{28}$$

$$u_t^s = \frac{s_t}{s_t + \overline{f}} \tag{29}$$

where the bar over the variable indicates the sample mean. Then to decompose variation in the unemployment rate he calculates the covariance of each measure (HP filtered) with the (HP filtered) one month ahead unemployment rate divided by the variance of the one month ahead unemployment rate.

Fujita & Ramey (2007) propose a different decomposition.[15] They begin with the steady state unemployment rate given above and note that one can take a log-linear approximation to obtain:

$$\begin{aligned} \Delta \ln u_t^{ss} &= (1 - u_{t-1}^{ss})\Delta \ln s_t - (1 - u_{t-1}^{ss})\Delta \ln f_t \\ \Delta u_t^{ss} &= \Delta s_t^r + \Delta f_t^r \end{aligned}$$

They then report the covariance of Δs_t^r and Δf_t^r divided by the variance of Δu^{ss}.

[15] A similar decomposition is found in Elsby *et al.* (2009)

Table 5: Unemployment Fluctuation Decompositions

	Shimer		Fujita and Ramey	
	Separation Rate	Finding Rate	Separation Rate	Finding Rate
Data	0.18	0.82	0.36	0.64
Model	0.29	0.69	0.36	0.64

This table contains the results of the Shimer and Fujita and Ramey decompositions for the data and from the model. For the model, simulations of length 3000 are run. And I keep the last 3*4*188 observations to match the 188 quarters of data. This table reports the median across all simulations.

Table 5 reports the results.[16] The Shimer decomposition gives 80% of the weight to finding rate fluctuations in explaining unemployment fluctuations while the Fujita and Ramey decomposition gives only 64% of the weight to find rate fluctuations. In my model I find using finding rate fluctuations only one is able to explain only 72% of the volatility of unemployment. Therefore my results are half way between the Fujita and Ramey and Shimer decompositions. The table also includes the results from carrying out the decompositions using data from my model. The model comes close to replicating the two decompositions, though understates the role of the finding rate in the Shimer decomposition because the model implies a higher correlation between the separation rate and the unemployment rate than in the data.

6 Conclusion

Current research evaluating the Mortensen-Pissarides model can be broadly placed into two categories. One set of models uses wage rigidity to create substantial volatility in the job-finding rate, but assumes constant separation rates. Another set of models allows for endogenous separation but omits wage rigidity and does not generate sufficient volatility in the job-finding rate. As a result, neither set of models is equipped to fully explain unemployment fluctuations. In this paper, I solve a version of the MP model with endogenous separation and wage rigidity. The model is consistent with the volatility of unemployment,

[16] As in the main section of the paper, I use a sample beginning in 1964 and ending in 2010.

the job-finding rate and the job-separation rate while explaining 74% of observed volatility in vacancies. The model also matched the volatility of the average wage and gave reasonable levels of the response of the wage of new hires to productivity and unemployment. I show that a version of the model where the job-separation rate is constant fails to fully explain the volatility of unemployment. Job separation rate volatility then is necessary to explain unemployment fluctuations.

There were a few key shortcomings of the model. First it was unable to fully match the volatility of vacancies. Second, it could not fully match the strong negative correlation between unemployment and vacancies. Perhaps extending the model to include on-the-job search would improve these shortcomings. Additionally, the model implies correlations with productivity that are substantially higher than in the data. Adding additional shocks to the model may make the model more realistic in this regard.

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