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# Endogenous separation, wage rigidity and the dynamics of unemployment



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## ABSTRACT

Previous attempts to evaluate the Mortensen–Pissarides model rely on *either* endogenous separation or wage rigidity. In this paper I simulate a version of the Mortensen–Pissarides (MP) model with wage rigidity *and* endogenous separation. The model is then able to answer a key question in the literature: can wage rigidity and endogenous separation explain the joint dynamics of unemployment, vacancies and wages? I find that it can. The model generates sufficient volatility in unemployment, the separation rate and the finding rate, 75% of the observed volatility in vacancies, and 70% of the Beveridge curve (the negative correlation between unemployment and vacancies). More substantially, the model matches the volatility of the average wage and generates a response of new hires' wages to productivity and unemployment consistent with key estimates in the literature. I then simulate the model while restricting the separation rate to be constant and show that the model predicts only 70% of the variance of unemployment. I conclude that finding rate fluctuations explain 70% of unemployment fluctuations halfway in between the most prominent estimates in the literature.

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## 1. Introduction

The Mortensen and Pissarides (1994) (MP) search and matching model is the dominant paradigm for studying unemployment fluctuations. Shimer (2005) argues that the model is unable to explain the volatility of unemployment, job-finding and vacancies. Subsequently a key question in the literature is: how can one modify the MP model to be consistent with the observed behavior of unemployment, vacancies and wages?

One strand of the literature (e.g. Hall, 2005a; Gertler and Trigari, 2009) attempts to rectify the model's shortcomings by allowing for wage rigidity. However this work assumes a constant separation rate. While earlier work (e.g. Shimer, 2012; Hall, 2005b) has argued that the separation rate is relatively acyclical and contributes little to unemployment fluctuations, much recent work has demonstrated a role for variation in the separation rate in explaining unemployment fluctuations. For example, Elsby et al. (2009) which concludes, "A complete understanding of cyclical unemployment requires an explanation of countercyclical inflow rates". Models with a constant separation rate, then, can only give an incomplete explanation for unemployment fluctuations. Additionally, these models cannot evaluate the required degree of wage rigidity since they must overstate finding rate volatility, and hence wage rigidity, in order to explain unemployment volatility.

Noting that the separation rate appears to be endogenous, another strand of the literature (e.g. Bils et al., 2011; Fujita and Ramey, 2012; Menzio and Shi, 2011) focuses on modeling endogenous separation. However these models suffer from the

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Shimer puzzle (Shimer, 2005) i.e. they generate too little variance in the job finding rate. For a thorough analysis of these two approaches and their implication for unemployment volatility see Mortensen and Nagypal (2007).

Both strands of the literature are important contributions to our understanding of unemployment fluctuations. However, since both approaches underestimate the importance of one channel in generating unemployment fluctuations (either job-finding or job-separation) neither can fully explain the volatility in unemployment fluctuations or evaluate the required degree of wage rigidity. I solve a version of the MP model with endogenous separation and wage rigidity. The model is able to explain the volatility of unemployment, the separation rate, and the finding rate. The model explains 74% the volatility of vacancies and 70% of the Beveridge Curve (the strong negative correlation between unemployment and vacancies). The model is consistent with the volatility of the average wage and more significantly, the model predicts a response of the wages of new hires to unemployment and productivity consistent with the main estimates in the literature.

I also show that restricting the separation rate to be constant allows the model to better explain the Beveridge curve and the volatility of vacancies, but the model cannot explain the observed volatility of unemployment. The constant separation rate model explains only 70% of the observed volatility of unemployment. This estimate is halfway between the two most prominent estimates of the contribution of the finding rate to unemployment fluctuations.

The paper makes several contributions to the literature. It is the first paper, to the best of my knowledge, to examine if endogenous separation and wage rigidity simultaneously can explain unemployment volatility. More significantly, it is the first to fully examine if the required degree of wage rigidity is counterfactual since models with a constant separation rate must necessarily overestimate the degree of wage rigidity needed to explain unemployment fluctuations. I find that the required degree of wage rigidity is in line with the key estimates in the literature (see Section 3.3 for more details). Finally, the paper uses a simple framework, with only one modification (wage rigidity) to the original MP model, in order to explain unemployment fluctuations.

The rest of the paper proceeds as follows. Section 2 describes the baseline model with endogenous separation and wage rigidity. Section 3 explains the model solution, calibration and gives the results from simulating the model. In Section 4, I demonstrate that once the separation rate is restricted to be constant, the model predicts too little variance in unemployment. In Section 5 I compare the model to other attempts to quantify the importance of the separation rate in explaining unemployment fluctuations. Section 6 examines the robustness of the results to alternate choices for model parameters and the idiosyncratic productivity distribution. Section 7 concludes.

## 2. Model

### 2.1. Theoretical model

#### 2.1.1. Informal description

In this Section I describe a version of the Mortensen and Pissarides (1994) model with wage rigidity. The model is a discrete time version of the Mortensen–Pissarides model. The model has large, persistent idiosyncratic productivity shocks to allow for endogenous separation. Some matches will receive a large enough negative productivity shock that the value of unemployment exceeds the value of production. These matches separate and the worker becomes unemployed.

My main departure from the standard MP model is the inclusion of wage rigidity. Shimer (2005) demonstrates that the Mortensen–Pissarides model does not generate sufficient unemployment volatility when workers' outside options are low. Finding this also to be the case for my model as well, I add wage stickiness via a wage norm, as in Hall (2005a), to increase the model's ability to generate unemployment volatility.

Because wages are rigid, and there are large idiosyncratic productivity shocks, wages may be, at times outside the bargaining set of the worker and firm. If this is the case, I assume that the outside option binds and that the wage adjusts to avoid an inefficient separation. Therefore, if the match receives a shock such that the wage is too high and the firm will want to sever the match, the wage adjusts so the firm's share of the surplus is zero. This makes the firm indifferent between keeping and firing the worker. Similarly if the wage is too low such that the worker would want to quit the match, I assume that the wage rises so that the worker gets a zero share of the surplus. I now proceed to a formal description of the model.

#### 2.1.2. Match productivity

At the beginning of the period there is a mass of worker–firm matches. Workers maximize expected discounted lifetime income. Firms maximize expected discounted profits. A fraction  $\rho^x$  of matches exogenously separates into unemployment.<sup>1</sup> Remaining firms produce according to the following production function  $x_{i,t}y_t$  where  $x_{i,t}$  is the idiosyncratic productivity level.  $x_{i,t} = x_{i,t-1}$  with probability  $1 - \phi$  and with probability  $\phi$ ,  $x_{i,t}$  is drawn from the discrete distribution  $H(x)$  with maximum value  $x^h$ .  $y_t$  represents aggregate productivity, which follows the AR(1) process  $\ln y_t = \rho^y \ln y_{t-1} + \varepsilon_t$  where  $\varepsilon_t$  is an i.i.d normal random variable with mean 0 and standard deviation  $\sigma^\varepsilon$ .

<sup>1</sup> Exogenous separation can be thought of as needing to leave a job for personal reasons or as receiving a permanent shock that destroys the value of the job.

### 2.1.3. Match surplus and separation

After observing the idiosyncratic and aggregate levels of productivity, the pairs calculate the expected surplus of remaining in the match:

$$S_t(x_{i,t}) = x_{i,t}y_t + G_t(x_{i,t}) - (U_t + b) \quad (1)$$

$G_t(x_{i,t})$  represents the expected future discounted value to the firm and the worker if they remain in a match with idiosyncratic productivity level  $x_{i,t}$ .  $G_t(x_{i,t})$  is increasing in  $x_{i,t}$  and varies positively with the state of aggregate productivity.  $U_t$  represents the future benefits that will accrue to the worker if she is unemployed this period, and  $b$  represents the flow value of being unemployed. Again  $U_t$  increases when aggregate productivity increases. Note that the surplus is the value of the match in excess of the worker's outside option, the value of unemployment. The firm's outside option is normalized to zero.

Since  $S_t(x_{i,t})$  is increasing in  $x_{i,t}$  there is a threshold value of idiosyncratic productivity ( $x_t^*$ ) below which the surplus is zero and the match is terminated:

$$0 = x_t^*y_t + G_t(x_t^*) - (U_t + b) \quad (2)$$

### 2.1.4. Wage setting

The standard MP model assumes that wages are perfectly flexible and adjust so that the firm gets a share  $\pi$  of the surplus. In this model I take the approach of Hall (2005a) and assume that wages are not perfectly flexible. Instead, wages are a weighted average between the wage that would give the firm a share  $\pi$  of the surplus and a wage norm. This assumption allows the firm's share of the surplus to vary over time. After a negative aggregate productivity shock, the wage does not adjust fully downward, therefore the firm must absorb most of the decline in aggregate productivity. This reduces their incentive to recruit and lowers the job-finding rate. This mechanism can generate additional volatility in job-finding. Wages then are given by:

$$w_t(x_i) = \lambda \bar{w}_i + (1 - \lambda)((1 - \pi)x_i y_t + \pi b + F_t(x_i) - \pi(G_t(x_i) - U_t)) \quad (3)$$

where  $F_t(x_i)$  is the future expected discounted payments that accrue to the firm in a match with idiosyncratic productivity  $x_i$ . This function is increasing in  $x_i$  and increases when the aggregate productivity level increases.  $(1 - \pi)x_i y_t + \pi b + F_t(x_i) - \pi(G_t(x_i) - U_t)$  is the wage that, when paid, would give the firm a share  $\pi$  of the total surplus.  $\bar{w}_i$  represents a wage norm that will be defined shortly.  $\lambda$  is a measure of wage stickiness. The closer  $\lambda$  is to one, the more rigid are wages.

Since wages are rigid, it is possible that the idiosyncratic productivity level is low enough that the firm would want to fire the worker when there is positive value in the match (i.e.  $x_i > x_t^*$ ). The firm would want to fire the worker if:

$$x_i y_t - w_t(x_i) + F_t(x_i) < 0 \quad (4)$$

Here the value of the match to the firm  $x_i y_t + F_t(x_i)$  is less than the wage it must pay. If the firm would want to fire the worker at the wage given by the wage rule, I assume that the wage adjusts so that the firm's share of the surplus is equal to zero, i.e. the firm is indifferent between keeping or firing the worker. In this case the wage is given by:

$$w_t(x_i) = x_i y_t + F_t(x_i) \quad (5)$$

It is also possible that the wage is too low and the worker would want to exit the match. This event would occur if:

$$w_t(x_i) + G_t(x_i) - F_t(x_i) - (U_t + b) < 0 \quad (6)$$

Here the value of the worker's outside option ( $U_t + b$ ) is greater than the value of the job to the worker: the wage  $w_t(x_i)$  plus the expected future discounted payments that accrues to the worker  $G_t(x_i) - F_t(x_i)$  (the joint firm and worker value minus the value that accrues to the firm). In this case I assume that the wage adjusts so that the worker is indifferent between quitting the job and staying. Then wages are given by:

$$w_t(x_i) = (U_t + b) - G_t(x_i) + F_t(x_i) \quad (7)$$

I assume that the wage norm is the wage that gives the firm a share  $\pi$  of the surplus in steady state (where  $y_t = 1$ ). Therefore:

$$\bar{w}_i = (1 - \pi)x_i + \pi b + F^{ss}(x_i) - \pi(G^{ss}(x_i) - U^{ss}) \quad (8)$$

### 2.1.5. Continuation value functions

To solve the model it is necessary to calculate the continuation value functions. The expected future payments of the match to the firm satisfy:

$$F_t(x_i) = \beta(1 - \rho^x)E_t \left[ (1 - \phi)[x_i y_{t+1} - w_{t+1}(x_i) + F_{t+1}(x_i)] * 1(x_i > x_{t+1}^*) \right. \\ \left. + \phi \int_{x_{t+1}^*}^{x_t^*} [\tilde{x}_i y_{t+1} - w_{t+1}(\tilde{x}_i) + F_{t+1}(\tilde{x}_i)] dH(\tilde{x}_i) \right] \quad (9)$$

The firm discounts future payments at a rate  $\beta$ , and the match remains with probability  $(1 - \rho^x)$ . With probability  $(1 - \phi)$ ,  $x_{i,t+1} = x_i$  and the match remains if  $x_{i,t+1} > x_{t+1}^*$  (hence the need for the indicator function  $1(\cdot)$ ). With probability  $\phi$  the match

receives a new value for  $x_i$ . If the idiosyncratic productivity shock is above  $x_{t+1}^*$ , the match produces. The firm collects  $\tilde{x}_i y_{t+1}$ , pays the worker  $w_{t+1}(\tilde{x}_i)$ , and the match has continuation value  $F_{t+1}(\tilde{x}_i)$  to the firm.

The total expected payments from remaining in the match today which accrue to either the firm or the worker satisfy:

$$G_t(x_i) = \beta E_t \left[ (1 - \rho^x) \left( (1 - \phi) \max(x_i y_{t+1} + G_{t+1}(x_i) - (U_{t+1} + b), 0) \right) + \phi \int_{x_{t+1}^*}^{x_i^h} [\tilde{x}_i y_{t+1} + G_{t+1}(\tilde{x}_i) - (U_{t+1} + b)] dH(\tilde{x}_i) \right] + U_{t+1} + b \quad (10)$$

In the event that the match does not separate exogenously, with probability  $(1 - \phi)$ ,  $x_{i,t+1} = x_i$  and the surplus of the match is given by  $\max(x_i y_{t+1} + G_{t+1}(x_i) - (U_{t+1} + b), 0)$ . With probability  $\phi$  the match receives a new value for  $x_i$ , and in the case  $\tilde{x}_i > x_{i,t+1}^*$  the match has surplus value  $\tilde{x}_i y_{t+1} + G_{t+1}(x_i) - (U_{t+1} + b)$ . In any event, the worker is guaranteed her outside option  $U_{t+1} + b$ .

Finally, the value of unemployment is:

$$U_t = \beta E_t \left[ \frac{m_t}{u_t} (1 - \rho^x) \left( (1 - \phi) [w_{t+1}(x^h) + G_{t+1}(x^h) - F_{t+1}(x^h) - (U_{t+1} + b)] \right) + \phi \int_{x_{t+1}^*}^{x^h} [w_{t+1}(x_i) + G_{t+1}(x_i) - F_{t+1}(x_i) - (U_{t+1} + b)] dH(x_i) \right] + U_{t+1} + b \quad (11)$$

The worker finds a job with probability  $\frac{m_t}{u_t}$  (the number of matches per unemployed worker) and with probability  $(1 - \rho^x)$  the match does not separate exogenously. I assume that all matches begin at the highest idiosyncratic productivity level, though the idiosyncratic productivity level can change before production.<sup>2</sup> Therefore, with probability  $(1 - \phi)$ ,  $x_i = x^h$  and the worker gets surplus  $w_{t+1}(x^h) + G_{t+1}(x^h) - F_{t+1}(x^h) - (U_{t+1} + b)$ . With probability  $\phi$  the match receives a new value for  $x_i$  and in the case  $x_{i,t+1} > x_{i,t+1}^*$  the worker surplus equals  $w_{t+1}(\tilde{x}_i) + G_{t+1}(\tilde{x}_i) - F_{t+1}(\tilde{x}_i) - (U_{t+1} + b)$ . In all cases, the worker receives her outside option  $U_{t+1} + b$ .

### 2.1.6. Employment dynamics

Workers separate from jobs into unemployment and I calculate flows from employment to unemployment as:

$$EU_{t+1} = \rho^x n_t + (1 - \rho^x) \left[ \phi H(x_{t+1}^*) n_t + (1 - \phi) \sum_{x_i < x_{t+1}^*} n_t(x_i) \right] \quad (12)$$

here  $n_t$  is the total number of matches which produce at time  $t$  and  $n_t(x_i)$  is the total number of matches that produce at production level  $x_i$ . A fraction  $\rho^x$  of all matches separate exogenously into unemployment each period. In addition, a fraction  $\phi H(x_{t+1}^*)$  will have idiosyncratic productivity next period below the separation threshold as will all those for which  $x_i$  remains less than  $x_{t+1}^*$ .

The separation rate is given by

$$s_{t+1} = \frac{EU_{t+1}}{n_t} \quad (13)$$

I assume that new matches begin at the highest productivity level ( $x^h$ ) but are subject to the idiosyncratic shock so that next period's employment stocks are given by:

$$n_{t+1}(x_i) = 0 \quad \text{if } x_i < x_{t+1}^* \quad (14)$$

$$n_{t+1}(x_i) = (1 - \rho^x) \phi h(x_i) (n_t + m_t) + (1 - \rho^x) (1 - \phi) n_t(x_i) \quad \text{if } x_{t+1}^* \leq x_i < x^h \quad (15)$$

$$n_{t+1}(x^h) = (1 - \rho^x) \phi h(x^h) (n_t + m_t) + (1 - \rho^x) (1 - \phi) (n_t(x^h) + m_t) \quad (16)$$

where  $h(x_i)$  is the probability that the draw from the distribution  $H(x) = x_i$  and  $m_t$  is the number of new matches this period. All matches with productivity less than  $x_{t+1}^*$  are destroyed. For matches with productivity between  $x_{t+1}^*$  and  $x^h$ , a fraction  $(1 - \rho^x)(1 - \phi)$  remain at the same productivity level and they are joined by  $(1 - \rho^x) \phi h(x_i) (n_t + m_t)$  who receive a new idiosyncratic productivity level equal to  $x_i$ . For matches with productivity  $x_i = x^h$  the calculation is the same, but we take into account that the new matches also begin at productivity level  $x^h$ .

Finally, unemployment evolves according to:

$$u_{t+1} = u_t + EU_{t+1} - (1 - \rho^x) [(1 - \phi) m_t + \phi (1 - H(x_{t+1}^*)) m_t] \quad (17)$$

and the size of the labor force is normalized to 1 so that  $n_t + u_t = 1$ . Here inflow into unemployment equals  $EU_{t+1}$  and outflow from unemployment equals the number of matches times the fraction that do not separate back into unemployment.

The next two equations determine the equilibrium number of matches and vacancies. Firms post vacancies up to the point where the marginal benefit of doing so equals the marginal cost  $c$ :

$$c = \frac{m_t}{v_t} F_t(x^h) \quad (18)$$

<sup>2</sup> I find this assumption convenient computationally. However, it is not necessary for the results.

$F_t(x^h)$  is the value today of filling a vacancy,  $v_t$  is the number of vacancies posted, and  $\frac{m_t}{v_t}$  is the likelihood that the vacancy is filled.

The following Cobb–Douglas function determines the number of matches:

$$m_t = Au_t^\alpha v_t^{1-\alpha} \quad (19)$$

$A$  is the matching efficiency parameter. This matching function exhibits constant returns to scale and  $m$  is increasing in  $u$  and  $v$ . Moreover,  $\frac{m}{v}$  and the finding rate,  $\frac{m}{u}$ , are decreasing in  $v$  and  $u$  respectively. This type of random matching function is meant to model frictions in the labor market. Unemployed workers cannot immediately find a job, but do so randomly with a probability less than one.

### 2.1.7. Timeline

To summarize:

1. At time  $t$  there is a vector  $n_t(x_i)$  of individuals employed and producing in each job and a number of unemployed  $u_t$ .
2. Firms post vacancies and the number of matches,  $m_t$ , is determined by the free entry condition (18) and the matching function (19).
3. The next level of aggregate productivity  $y_{t+1}$  is drawn and then a fraction  $\phi$  of all matches (new and old) receive a new value of  $x_i$  drawn from the distribution  $H(x)$ .
4. A fraction of all matches  $\rho^x$  separate into unemployment.
5. All matches for which  $x_i < x_{t+1}^*$  separate into unemployment.
6. Remaining matches produce and firms post vacancies to determine  $m_{t+1}$ .

### 2.1.8. Equilibrium

The equilibrium of the model is a set of functions  $F(x_i, y_i)$ ,  $G(x_i, y_i)$ ,  $U(y_i)$  which are fixed points of the recursive value function Eqs. (9)–(11).  $F(x_i, y_i)$  and  $G(x_i, y_i)$  are mappings from  $\mathbb{R}^2 \rightarrow \mathbb{R}$  and increasing in their arguments.  $U(y_i)$  is a mapping from  $\mathbb{R} \rightarrow \mathbb{R}$  and increasing in its argument. The equilibrium also requires a vector  $x^*(y_i)$  solving the separation threshold Eq. (2), a wage setting function,  $w(x_i, y_i)$  satisfying the wage setting equations: (3), (5) and (7), and a vector of market tightness  $\theta(y_i) \equiv \frac{v_t}{u_t}$  satisfying the free entry condition (18) and the matching function (19).  $x^*(y_i)$  and  $\theta(y_i)$  are mappings from  $\mathbb{R} \rightarrow \mathbb{R}$ .  $\theta(y_i)$  is increasing in its argument and  $x^*(y_i)$  is decreasing in its argument.  $w(x_i, y_i)$  is a mapping from  $\mathbb{R}^2 \rightarrow \mathbb{R}$  and generally (weakly) increasing in its arguments.

## 2.2. Empirical motivation for wage rigidity

Real-wage rigidity is an important feature of the model in this paper. As pointed out by Shimer (2005), the standard Mortensen–Pissarides model does not generate sufficient volatility in unemployment and vacancies. Hall (2005a) notes that the model's amplification mechanisms are greatly improved by adding real wage rigidity. This is true even if the wage is allowed to adjust to avoid inefficient separations. Substantial real-wage rigidity moves the model towards paying the worker a wage that varies less with the state of aggregate productivity. As a result, the firm keeps gains from aggregate-productivity increases and absorbs losses from aggregate-productivity decreases. This mechanism makes the firm's recruiting incentives more procyclical, generating more variance in unemployment and vacancies through the finding rate.

Beyond the empirical necessity, additional research points to the importance of real wage rigidity. Hall (2005a) argues that there is a social consensus as to what the fair wage is and that a sense of a fair wage may affect wage setting. Akerlof et al. (1996) and Bewley (1999) support this view as well. Falk et al. (2006) introduce minimum wages in experimental settings. They find introducing minimum wages raises reservation wages. Even after removing the minimum wage, the reservation wages remain higher than before. They argue that the minimum wage shapes what subjects consider a fair wage.

## 2.3. Dynamics of new hires' wages

While the average wage's relative acyclical is well known, the cyclical nature of new hires' wages is currently an active research area. As noted by Pissarides (2009) and Haefke et al. (2008), in a sample of those who have begun work recently, wages are sensitive to changes in aggregate productivity. For example, Haefke et al. (2008) find that a 1% increase in productivity leads to a 0.79% increase in the wage of new hires. This evidence would seem to cast doubt on the ability of wage rigidity to explain fluctuations in job-finding.

However, as Gertler and Trigari (2009) argue, these studies fail to control for changes in the type of job at which workers work. For example, if in recessions workers transition more from well paying jobs (e.g. manufacturing) to poorer paying jobs (e.g. retail), wages of new hires will be very sensitive to aggregate productivity. After controlling for job-specific characteristics, they find that wages of new hires are no more sensitive to the aggregate state of the economy than those of current employees.

Recent evidence from Portugal attempts to address the argument of Gertler and Trigari. Carneiro et al. (2012) estimate a wage equation with firm and worker fixed effects and find that the cyclical nature of the wage for new hires is twenty percent

**Table 1**  
Endogenous separation model parameters.

Parameter	Symbol	Value	Source
Discount factor	$\beta$	0.99 <sup>1/12</sup>	Standard
Productivity autoregressive parameter	$\rho^z$	0.96 <sup>1/12</sup>	Standard
Aggregate productivity shock stdev.	$\sigma^e$	0.002605	Stdev. of productivity
Exogenous separation rate	$\rho^x$	0.00542	Silva and Toledo (2009)
Flow value of unemployment	$b$	0.75	Costain and Reiter (2008)
Sticky wage weight	$\lambda$	0.924	Stdev. of finding rate
Matching function elasticity	$\alpha$	0.5246	Hosios (1990)
Probability of new idiosyncratic shock	$\varphi$	0.168	Stdev. of separation rate
Vacancy posting cost	$c$	0.3741	The next five parameters are calibrated by matching: mean finding rate (Shimer, 2005), mean firm matching rate (Davis et al., 2009), mean separation rate (Hall, 2005b), recruiting cost (Hall and Milgrom, 2008), mean productivity equal to 1
Matching function efficiency	$A$	0.1621	
Firm's bargaining share	$\pi$	0.4754	
Idiosyncratic standard deviation	$\sigma^z$	0.2689	
Highest idiosyncratic productivity	$x^h$	1.2077	

This table describes the parameters from the main model in the paper and explains how the variables are calibrated. It also gives the values of the calibrated parameters.

higher than the cyclical variation of the wage for those continuing in the same job. However, Martins et al. (2012), discuss concerns about the economic interpretation of Carneiro et al.'s (2012) results given the small amount of residual variation remaining after the use of their various fixed effects. They construct a time series of entry jobs and find a 1 point increase in the unemployment rate corresponds to a 1.8% decrease in the entry level job wage.

For my model I find that a 1 percentage point increase in the unemployment rate corresponds to a 1.1% decrease in the new hire wage, within the confidence interval of the Martins et al. (2012) estimate. Similarly, I find that the wage of new hires increases by 0.7% when aggregate productivity increases by 1%, only slightly below the estimates of Haefke et al. (2008).<sup>3</sup> Hence, the wage stickiness in my model is consistent with the estimates on the cyclical variation of the wages of new hires.

Since the model adjusts wages which are outside the bargaining set one might ask how much of the wage variation comes from these changes. In my simulation I also track what the wage would have been if it had not adjusted. The average non-adjusted wage is 90% as volatile as the adjusted wage, so these changes are not substantial quantitatively. Another issue of concern may be that the model's adjustment of wages to avoid an inefficient separation may result in lower volatility for the wages of new hires versus the average wage. However, I find that the wages of new hires is 94% as volatile as the average wage lessening this concern.

### 3. Model solution, estimation, and results

#### 3.1. Model Solution

To solve the model I discretize aggregate productivity  $\ln y$  using the method of Tauchen and Hussey (1991). This method gives a grid of possible values for  $\ln y$ ,  $y = \{y_1, \dots, y_n\}$  and a transition matrix  $\Pi_{ij} = \text{Prob}(y_{t+1} = y_i | y_t = y_j)$ . I set  $n = 21$ . I then guess initial functions  $F(x_i, y)$ ,  $G(x_i, y)$ , and  $U(y)$  and iterate on Eqs. (9)–(11) until the functions converge.<sup>4</sup>

To approximate the distribution for the idiosyncratic shocks I follow Fujita and Ramey (2012) and let  $x_k = \{x_1, \dots, x_K\}$  with  $x_K = x^h$ . I let  $x_1 = 1/K$ ,  $x_k - x_{k-1} = \frac{x^h}{K}$  and set  $\text{Prob}(x_i = x_j) = \frac{x^h \log \text{norm}(x_j, 0, \sigma^z)}{K}$  for  $j = 1, \dots, K-1$  and  $\text{Prob}(x_i = x_K) = 1 - \sum_{j=1}^{K-1} \text{Prob}(x_i = x_j)$ .<sup>5</sup>  $\log \text{norm}(x_j, 0, \sigma^z)$  denotes the lognormal p.d.f. evaluated at  $x_j$  with mean zero and standard deviation  $\sigma^z$ . I set  $K = 200$ .

<sup>3</sup> See Section 3.5 for details of this calculation.

<sup>4</sup> As common in the literature, I look for an equilibrium where the value function depends only on idiosyncratic and aggregate productivity and not the employment distribution. Existence follows by construction however I am unable to demonstrate uniqueness.

<sup>5</sup> Fujita and Ramey (2012) uses  $\frac{\log \text{norm}(x_j, 0, \sigma^z)}{K}$  instead of  $\frac{x^h \log \text{norm}(x_j, 0, \sigma^z)}{K}$  though I find the difference to be unimportant.

**Table 2**  
Endogenous separation model results.

	$\sigma(u)$	$\sigma(v)$	$\sigma(f)$	$\sigma(s)$	$\sigma(w)$	$\sigma(p)$
Data	0.118	0.136	0.082	0.052	0.011	0.013
Model	0.13	0.1	0.082	0.052	0.011	0.013
	$\rho(u, v)$	$\rho(u, f)$	$\rho(u, s)$	$\rho(v, f)$	$\rho(v, s)$	$\rho(f, s)$
Data	-0.91	-0.94	0.41	0.86	-0.52	-0.21
Model	-0.64	-0.92	0.87	0.89	-0.78	-0.9
	$\rho(u, p)$	$\rho(f, p)$	$\rho(s, p)$	$\rho(v, p)$	$\rho(w, p)$	
Data	-0.24	0.18	-0.57	0.44	0.55	
Model	-0.91	0.99	-0.92	0.88	0.99	
Response of new hire wage to		Unemployment		Productivity		
		Data	Model	Data	Model	
		-1.8	-1.1	0.79	0.7	
		(0.4)		(0.4)		

This table contains the results of simulating the main model in the paper. Simulations of length 3000 are run. And I keep the last  $3 * 4 * 188$  observations to match the 188 quarters of data. This table reports the median across all simulations.

### 3.2. Calibration

Table 1 contains the parameters. The model is calibrated at a weekly frequency and I assume four weeks per month. I set the discount factor  $\beta = 0.99^{1/2}$  and the productivity autocorrelation parameter  $\rho^z = 0.96^{1/2}$ .<sup>6</sup> Following Silva and Toledo (2009) I set the exogenous separation probability to  $\rho^x = 0.00542$  which corresponds to their quarterly probability of 0.065. I then set the standard deviation of the productivity shocks  $\sigma^z = 0.002605$  in order to match the standard deviation of HP-filtered aggregate labor productivity.<sup>7</sup> As suggested by Costain and Reiter (2008) I set  $b = 0.75$  to be consistent with the response of unemployment to changes in the unemployment insurance replacement ratio.<sup>8</sup> I set  $\lambda = 0.924$  to match the standard deviation of the finding rate and  $\phi = 0.168$  to match the standard deviation of the separation rate. I set  $\alpha = 1 - \pi$  (the worker's bargaining share) the so called Hosios (1990) condition. Finally  $c = 0.3741$ ,  $A = 0.1621$ ,  $\chi^h = 1.2077$ ,  $\sigma^z = 0.2689$ , and  $\pi = 0.4754$ . To calibrate these parameters I match the following moments in steady state. I normalize mean productivity across jobs  $\left(\frac{\sum_i \chi_i + \pi(\chi_i)}{\sum_i \pi(\chi_i)}\right) = 1$ . I set the steady state finding rate to 0.12 which implies a monthly finding rate of 0.40 consistent with the evidence in Shimer (2005). I set the overall weekly separation rate to 0.0082 which translates to a monthly separation rate of 0.0323 consistent with the estimate in Hall (2005b). I set the weekly vacancy filling probability  $\left(\frac{\pi}{v}(1 - \rho^x)[(1 - \phi) + \phi(1 - H(x^*))]\right)$  equal to 0.226 which corresponds to a daily vacancy filing rate of 5% as in Davis et al. (2009). And I set the steady state job filling cost  $\frac{c}{m/v} = 14\%$  the quarterly wage as in Hall and Milgrom (2008).

### 3.3. Model simulation

To simulate the model I run 1000 trials beginning at the steady state level of productivity and employment. I simulate 3000 weeks and I keep only the last  $188 * 3 * 4 = 2256$  weeks of data to match the length of my data sample. I average the data at the quarterly level and HP-filter the log variables with a smoothing parameter of 1600.<sup>9</sup> I then report the median values across the 1000 trials.

### 3.4. Data

The data are as in Shimer (2005) except data begin in 1964 and end in 2010.<sup>10</sup> The measure of unemployment is the seasonally adjusted civilian unemployment rate from the Bureau of Labor Statistics (BLS). The series on vacancies is the Conference Board's Help Wanted advertisements series.<sup>11</sup> Productivity is measured as real output per person in the non-farm business sec-

<sup>6</sup> At a quarterly frequency it is common to assume an AR(1) parameter of 0.96, see for example Silva and Toledo (2009). This motivates my choice of  $0.96^{1/2}$ . I find that in my model the autocorrelation of quarterly productivity is 0.74 versus 0.79 in the data. However, since I HP-filter the quarterly data with a smoothing parameter of 1600, I cannot match the autocorrelation of 0.79 in the data even with higher levels of weekly persistence. This is because the quarterly averaging and the HP-filtering results in a persistence below the data even as  $\rho^z \rightarrow 1$ . As an additional robustness check, I find that adopting the weekly value of 0.99 from Fujita and Ramey (2012) did not change the results.

<sup>7</sup> Aggregate productivity is given by  $\frac{y_t \sum_i \chi_i + \pi(\chi_i)}{\sum_i \pi(\chi_i)}$  where  $n_i(\chi_i)$  is employment in job  $i$  at time  $t$ .

<sup>8</sup> As in Costain and Reiter I find the steady state semi-elasticity of unemployment  $\frac{\partial \ln u_t}{\partial b} = 2$  in my model when  $b = 0.75$ .

<sup>9</sup> To compare the model to the data I convert the weekly model probabilities to monthly probabilities. Therefore, I set  $f^m = 1 - (1 - f^w)^4$  and  $s^m = 1 - (1 - s^w)^4$ . Similar results were obtained comparing weekly model transition rates to monthly rates calculated as in Shimer (2012).

<sup>10</sup> Wage data begin only in 1964.

<sup>11</sup> I thank Ken Goldstein at the Conference Board for providing me the data through 2010.

tor and come from the BLS Major Sector Productivity and Costs Program. The measure of wages is the BLS series on average hourly earning of production and non-supervisory employees in the private sector deflated with the consumer price index (CPI). This is the wage data used in Gertler and Trigari (2009). I follow the methodology of Shimer (2012) (Section 2) to construct series for the monthly job-finding probabilities and job-separation probabilities from the BLS's Current Population Survey (CPS) through 2010.<sup>12</sup> All data except for the productivity data are quarterly averages of monthly series. All data are transformed in logarithms and HP filtered with a smoothing parameter 1600.

### 3.5. Endogenous separation model results

Table 2 contains the main moments in the data. The results are comparable to Shimer (2005) though I use a lower, more conventional smoothing parameter (1600 versus  $10^5$ ) so the standard deviations are smaller.<sup>13</sup> The standard deviation of unemployment is about 12% and it is 13.6% for vacancies. The finding rate with a standard deviation of 8% is more volatile than the separation rate which has a standard deviation of 5%. The standard deviation for wages and productivity is about 1%. There is a strong negative correlation between unemployment and vacancies (the Beveridge curve) and unemployment and the finding rate. The correlation between the separation rate and unemployment is lower as is the correlation between the separation rate and the finding rate. The correlations with productivity have all the expected signs. However, in the last few years of the sample the correlation between the finding rate, unemployment and productivity has changed signs resulting in a low average correlation between these variables.

Table 2 also contains the results from simulating the model. Recall that the parameters of the model are chosen so that the model exactly matches the observed standard deviation of the job-finding rate, the separation rate and productivity.

For the moments I do not target with the calibration the model also performs well. The model predicts a standard deviation of unemployment equal to 0.13 close to the 0.118 in the data. The model explains 74% the volatility of vacancies. The model is able to do this while also exactly matching the volatility of wages. This is not by construction. Wage stickiness is calibrated to match the volatility of the finding rate; that the model then gives realistic volatility for the wage rate is remarkable. In the second half of the table I investigate the model's predictions for the response of new hires' wages to unemployment and productivity.<sup>14</sup> I find that the model predicts a 1.1% decrease in wages given a 1% increase in unemployment. This is within the confidence interval given by Martins et al. (2012). (Standard errors are given in parentheses.) Similarly the response of wages to aggregate productivity is 0.7, close to the estimates of 0.79 in Haefke et al. (2008). Wage stickiness is calibrated only to match the volatility of the finding rate. The model then shows that the degree of wage rigidity required by the MP model to match unemployment volatility is in line with the estimates of wage rigidity (at the micro-level) calculated in the literature.

The model also predicts all the correct signs for the correlation coefficients. It comes very close to matching the correlation of unemployment and the finding rate ( $-0.92$  versus  $-0.94$  in the data), the correlation of vacancies and the finding rate ( $0.89$  versus  $0.86$  in the data). The model overestimated the correlation between unemployment and the separation rate ( $0.87$  versus  $0.41$  in the data), the correlation between vacancies and the separation rate ( $-0.78$  versus  $-0.52$  in the data), and the correlation between the separation rate and the finding rate ( $-0.9$  versus  $-0.64$  in the data). Finally, the model cannot fully match the Beveridge curve. It predicts the correlation between unemployment and vacancies should be  $-0.64$  versus  $-0.91$ . Many models with endogenous separation predict a positive correlation between unemployment and vacancies as the increase in separations causes both an increase in unemployment and vacancies. That is not the case here because the presence of wage rigidity results in a volatile finding rate that counteracts this effect.

For the correlations with productivity, the model replicates the correct signs. However, it implies correlations with productivity that are too high because productivity is the only shock in the model.

It appears then that the standard MP model does a good job capturing the main moments in the data once it is allowed to have reasonable levels of wage rigidity and an endogenous separation rate. And it can do so without counterfactually high levels of wage rigidity.

## 4. Constant separation rate model

This section examines the model without an endogenous separation rate. Doing so demonstrates the need for endogenous separation to explain the volatility of unemployment and also gives an answer to the question of what percent of unemployment fluctuations come from changes in the separation rate. In the next section I compare my answer with other answers provided in the literature.

<sup>12</sup> This method uses data on unemployment and short term unemployment to infer the job-finding and job-separation probabilities correcting for time aggregation bias. The method also requires data from the incoming rotation groups to correct for changes in the CPS survey design beginning in 1994.

<sup>13</sup> Using a smoothing parameters of  $10^5$  did not change the evaluation of the model's fit to the data or the main conclusions of the paper.

<sup>14</sup> These values are calculated as the regression coefficient  $\beta$  in  $\Delta \ln w_t^h = \alpha + \beta x_t + \varepsilon_t$ . Where  $w_t^h$  is the new hire wage.  $\beta = \frac{\text{cov}(x_t, w_t^h)}{\text{var}(x_t)}$ .  $x_t$  is the change in the unemployment rate or the change in log aggregate productivity defined in Section 3.2. The new hire wage, taking account the probability of productivity changing immediately after hiring, is given by:  $[(1 - \phi)w(x^h) + \bar{w}_i\phi(1 - H(x^*))]/(1 - \phi H(x^*))$  where  $\bar{w}_i$  is the average wage of those who change.

**Table 3**

Parameters in the constant separation rate model.

Parameter	Symbol	Value	Source
Discount factor	$\beta$	0.99 <sup>1/12</sup>	Standard
Productivity autoregressive parameter	$\rho^z$	0.96 <sup>1/12</sup>	Standard
Aggregate productivity shock stdev.	$\sigma^\phi$	0.00239	Stdev. of productivity
Exogenous separation rate	$\rho^x$	0.00817	Hall (2005b)
Flow value of unemployment	$b$	0.75	Costain and Reiter (2008)
Sticky wage weight	$\lambda$	0.93125	Stdev. of finding rate
Matching function elasticity	$\alpha$	0.5238	Hosios (1990)
Vacancy posting cost	$c$	0.374	The next three parameters are calibrated by matching: mean finding rate (Shimer, 2005), mean firm matching rate (Davis et al., 2009), recruiting cost (Hall and Milgrom, 2008)
Matching function parameter	$A$	0.1622	
Firm's bargaining share	$\pi$	0.4762	

This table describes the parameters from the constant separation model in the paper and explains how the variables are calibrated. It also gives the values of the calibrated parameters.

To study these questions I remove the idiosyncratic productivity shocks. Now for all matches  $x_{i,t} = 1$ . Again, I allow wages to be rigid. Now wages are given by<sup>15</sup>:

$$w_t = \lambda \bar{w} + (1 - \lambda)((1 - \pi)y_t + \pi b + F_t - \pi(G_t - U_t)) \quad (20)$$

Again the wage norm is the wage in steady state (where productivity  $y$  is normalized to 1) that would give the firm a share  $\pi$  of the surplus.

$$\bar{w} = (1 - \pi) + \pi b + F^{ss} - \pi(G^{ss} - U^{ss}) \quad (21)$$

The continuation values are given by:

$$F_t = \beta(1 - \rho^x)E_t[y_{t+1} - w_{t+1} + F_{t+1}] \quad (22)$$

$$G_t = \beta E_t[(1 - \rho^x)(y_{t+1} + G_{t+1} - (U_{t+1} + b)) + U_{t+1} + b] \quad (23)$$

$$U_t = \beta E_t \left[ \frac{m_t}{u_t} (1 - \rho^x)(w_{t+1} + G_{t+1} - F_{t+1} - (U_{t+1} + b)) + U_{t+1} + b \right] \quad (24)$$

Employment and unemployment evolve according to:

$$n_{t+1} = (1 - \rho^x)(n_t + m_t) \quad (25)$$

$$u_{t+1} = u_t + \rho^x n_t - (1 - \rho^x)m_t \quad (26)$$

And the free entry condition becomes

$$c = \frac{m_t}{v_t} F_t \quad (27)$$

Calibration of the constant separation rate model is very similar to calibration of the endogenous separation rate model and summarized in Table 3. However I no longer need to calibrate the parameters associated with the idiosyncratic productivity process:  $\phi$  (the probability of a new shock),  $\sigma^z$  (the standard deviation of the idiosyncratic productivity process) and  $x^h$  (the highest level of productivity). Also, I set the exogenous separation rate  $\rho^x$  to 0.0082 consistent with an overall monthly separation rate of 0.0323.

#### 4.1. Constant separation results

The model with a constant separation rate is no longer able to match the volatility of unemployment. It predicts a volatility of 0.085 versus 0.118 in the data (see Table 4). However, it almost completely explains the volatility of vacancies (0.134 versus 0.136 in the data). This result is consistent with the analysis by Mortensen and Nagypal (2008) and Pissarides (2009). As in their work, I find that the constant separation rate model creates more volatility of vacancies than the endogenous separation rate model. In the endogenous separation rate model, additional separations during recessions leads to more demand for vacancies which mutes the fall in vacancies that normally occurs in recessions. This mechanism is not present in the constant separation rate model. The model also improves on the fit of the Beveridge curve (−0.79 versus −0.91 in

<sup>15</sup> I find without the idiosyncratic productivity shocks the wage rule results in wages always in the bargaining set. Therefore, there is no need to adjust the wage in this case.

**Table 4**

Constant separation rate model results.

	$\sigma(u)$	$\sigma(v)$	$\sigma(f)$	$\sigma(s)$	$\sigma(w)$	$\sigma(p)$
Data	0.118	0.136	0.082	0.052	0.011	0.013
Model	0.085	0.134	0.082	–	0.01	0.013
	$\rho(u, v)$	$\rho(u, f)$	$\rho(u, s)$	$\rho(v, f)$	$\rho(v, s)$	$\rho(f, s)$
Data	–0.91	–0.93	0.41	0.86	–0.52	–0.21
Model	–0.79	–0.91	–	0.96	–	–
	$\rho(u, p)$	$\rho(f, p)$	$\rho(s, p)$	$\rho(v, p)$	$\rho(w, p)$	
Data	–0.24	0.18	–0.57	0.44	0.55	
Model	–0.91	0.98	–	0.95	0.99	

This table contains the results of simulating the constant separation rate model in the paper. Simulations of length 3000 are run. And I keep the last  $3 * 4 * 188$  observations to match the 188 quarters of data. This table report the median across all simulations.

**Table 5**

Unemployment fluctuation decompositions.

	Shimer		Fujita and Ramey	
	Separation rate	Finding rate	Separation rate	Finding rate
Data	0.18	0.82	0.36	0.64
Model	0.29	0.69	0.36	0.64

This table contains the results of the Shimer and Fujita and Ramey decompositions for the data and from the model. For the model, simulations of length 3000 are run. And I keep the last  $3 * 4 * 188$  observations to match the 188 quarters of data. This table reports the median across all simulations.

the data), and matches almost exactly the correlation between the finding rate and unemployment and vacancies. The correlations with productivity are very similar to the model with endogenous separation.

This section demonstrates then that: 1. endogenous separation rates are necessary to match the volatility of unemployment however 2. endogenous separation makes it somewhat harder for the model to explain the volatility of vacancies and the negative correlation between unemployment and vacancies.

## 5. Comparison to other decompositions

The model then gives an answer to what percent of unemployment fluctuations are due to variations in the separation rate. The literature gives two other answers to this question from using data on unemployment fluctuations, separation rates and finding rates. I describe each of these decompositions and compare those conclusions to the conclusion of my model.

Shimer (2012) begins with the quarterly average of monthly observations on the one month ahead unemployment rate, separation rate and job-finding rate. He then notes that the steady state unemployment rate is given by  $u^{ss} = s/(s + f)$  where  $s$  is the separation rate and  $f$  is the job-finding rate. He creates two hypothetical measures of the unemployment rate:

$$u_t^f = \frac{\bar{s}}{\bar{s} + f_t} \quad (28)$$

$$u_t^s = \frac{s_t}{s_t + \bar{f}} \quad (29)$$

where the bar over the variable indicates the sample mean. Then to decompose variation in the unemployment rate he calculates the covariance of each measure (HP filtered) with the (HP filtered) one month ahead unemployment rate divided by the variance of the one month ahead unemployment rate.

Fujita and Ramey (2009) propose a different decomposition.<sup>16</sup> They begin with the steady state unemployment rate given above and note that one can take a log-linear approximation to obtain:

$$\begin{aligned} \Delta \ln u_t^{ss} &= (1 - u_{t-1}^{ss}) \Delta \ln s_t - (1 - u_{t-1}^{ss}) \Delta \ln f_t \\ \Delta u_t^{ss} &= \Delta s_t^r + \Delta f_t^r \end{aligned}$$

They then report the covariance of  $\Delta s_t^r$  and  $\Delta f_t^r$  divided by the variance of  $\Delta u_t^{ss}$ .

Table 5 reports the results of carrying out the Shimer and Fujita and Ramey decompositions using my data.<sup>17</sup> The Shimer decomposition gives 80% of the weight to finding rate fluctuations in explaining unemployment fluctuations while the Fujita and Ramey decomposition gives only 64% of the weight to find rate fluctuations. In my model I find, when using only finding

<sup>16</sup> A similar decomposition is found in Elsby et al. (2009)

<sup>17</sup> As in the main section of the paper, I use a sample beginning in 1964 and ending in 2010.

**Table 6**  
Robustness to exogenous separation probability.

$\rho^x = 0.05/12$						
Model	$\sigma(u)$ 0.126	$\sigma(v)$ 0.103	$\sigma(f)$ 0.082	$\sigma(s)$ 0.052	$\sigma(w)$ 0.011	$\sigma(p)$ 0.013
Model	$\rho(u,v)$ -0.67	$\rho(u,f)$ -0.92	$\rho(u,s)$ 0.8	$\rho(v,f)$ 0.9	$\rho(v,s)$ -0.68	$\rho(f,s)$ -0.8
Model	$\rho(u,p)$ -0.93	$\rho(f,p)$ 0.99	$\rho(s,p)$ -0.84 Unemp.	$\rho(v,p)$ 0.88 Prod.	$\rho(w,p)$ 0.99	
Response of new hire wage to			-1.25	0.73		
$\rho^x = 0.04/12$						
Model	$\sigma(u)$ 0.123	$\sigma(v)$ 0.106	$\sigma(f)$ 0.082	$\sigma(s)$ 0.052	$\sigma(w)$ 0.011	$\sigma(p)$ 0.013
Model	$\rho(u,v)$ -0.68	$\rho(u,f)$ -0.92	$\rho(u,s)$ 0.74	$\rho(v,f)$ 0.91	$\rho(v,s)$ -0.61	$\rho(f,s)$ -0.72
Model	$\rho(u,p)$ -0.93	$\rho(f,p)$ 0.99	$\rho(s,p)$ -0.78 Unemp.	$\rho(v,p)$ 0.88 Prod.	$\rho(w,p)$ 0.99	
Response of new hire wage to			-1.22	0.73		

This table contains the results of simulating the main model of the paper with alternative values for the exogenous separation probability ( $\rho^x$ ).

**Table 7**  
Robustness to flow value of unemployment.

$b = 0.65$						
Model	$\sigma(u)$ 0.112	$\sigma(v)$ 0.169	$\sigma(f)$ 0.082	$\sigma(s)$ 0.052	$\sigma(w)$ 0.01	$\sigma(p)$ 0.013
Model	$\rho(u,v)$ -0.64	$\rho(u,f)$ -0.85	$\rho(u,s)$ 0.64	$\rho(v,f)$ 0.95	$\rho(v,s)$ -0.09	$\rho(f,s)$ -0.3
Model	$\rho(u,p)$ -0.84	$\rho(f,p)$ 0.98	$\rho(s,p)$ -0.34 Unemp.	$\rho(v,p)$ 0.93 Prod.	$\rho(w,p)$ 0.99	
Response of new hire wage to			-1.05	0.54		
$b = 0.55$						
Model	$\sigma(u)$ 0.0925	$\sigma(v)$ 0.2324	$\sigma(f)$ 0.082	$\sigma(s)$ 0.052	$\sigma(w)$ 0.009	$\sigma(p)$ 0.013
Model	$\rho(u,v)$ -0.65	$\rho(u,f)$ -0.79	$\rho(u,s)$ 0.38	$\rho(v,f)$ 0.98	$\rho(v,s)$ 0.3	$\rho(f,s)$ 0.14
Model	$\rho(u,p)$ -0.82	$\rho(f,p)$ 0.98	$\rho(s,p)$ 0.04 Unemp.	$\rho(v,p)$ 0.95 Prod.	$\rho(w,p)$ 0.99	
Response of new hire wage to			-0.94	0.48		

This table contains the results of simulating the main model of the paper with alternative values for the flow value of unemployment ( $b$ ).

rate fluctuations, one is able to explain 72% of the volatility of unemployment. Therefore my results are halfway between the Fujita and Ramey and Shimer decompositions. The table also includes the results from carrying out the decompositions using data generated from my model. The model comes close to replicating the two decompositions, though understates the role of the finding rate in the Shimer decomposition because the model implies a higher correlation between the separation rate and the unemployment rate than in the data.

## 6. Robustness

In this section I examine the robustness of the results to alternative choices for some of the parameters and alternative assumptions about the distribution of idiosyncratic productivity shocks. First, I examine lower values of  $\rho^x$  the exogenous separation rate. This analysis will allow for a larger share of the separations to be endogenously generated from adverse productivity shocks. Second, I examine lower values of  $b$ , the flow value of unemployment, to see if the results are sensitive to the size of the worker's outside option. As there is a range of admissible values for  $b$  in the literature it is useful to know how the model behaves for alternative calibrations. Third, I examine a uniform distribution as an alternative to the log normal distribution for idiosyncratic productivity shocks.

**Table 8**  
Robustness to idiosyncratic productivity distribution.

<i>Uniform distribution</i>						
Model	$\sigma(u)$ 0.127	$\sigma(v)$ 0.106	$\sigma(f)$ 0.082	$\sigma(s)$ 0.052	$\sigma(w)$ 0.0107	$\sigma(p)$ 0.013
Model	$\rho(u,v)$ -0.58	$\rho(u,f)$ -0.9	$\rho(u,s)$ 0.86	$\rho(v,f)$ 0.88	$\rho(v,s)$ -0.58	$\rho(f,s)$ -0.8
Model	$\rho(u,p)$ -0.91	$\rho(f,p)$ 0.99	$\rho(s,p)$ -0.85	$\rho(v,p)$ 0.85	$\rho(w,p)$ 0.99	
Response of new hire wage to			Unemp -1.27	Prod. 0.734		

This table contains the results of simulating the main model of the paper with a uniform distribution for the idiosyncratic productivity distribution.

### 6.1. Exogenous separation rate

Table 6 displays the simulation results for using different assumptions for the exogenous separation rate  $\rho^x$ . The benchmark model sets  $\rho^x = 0.065/12$ . To allow for a larger share of separations to be endogenous I examine  $\rho^x = 0.05/12$  and  $\rho^x = 0.04/12$ . The rest of the calibration is the same as for the benchmark model (the main model of the paper). The results for the lower values of  $\rho^x$  are quite similar to results for the benchmark model. The model explains the volatility of unemployment and most of the volatility of vacancies. It still can match the volatility of the finding rate and separation rate while generating a realistic Beveridge curve. Importantly, the model still is able to match the volatility of wages and generates very similar dynamics for the new hire wage. An increase in unemployment leads to a -1.2% decrease in the new hire wage (it was -1.1% in the benchmark model) and an increase in productivity leads to a 0.7% increase in the new hire wage the same as in the benchmark model. Increasing the percentages of separations which are endogenous has resulted in a separation rate which is slightly less correlated with unemployment and productivity.<sup>18</sup> Though the difference is not large.

### 6.2. Flow value of unemployment

Table 7 contains the results allowing different values for the flow value of unemployment ( $b$ ). The benchmark model uses  $b = 0.75$ . Here I examine results for  $b = 0.65$  and  $b = 0.55$ . The model is still able to match the volatility of the finding rate and separation rate. The model matches the volatility of unemployment for  $b = 0.65$  but explains only 80% of the volatility of unemployment when  $b = 0.55$ . It is well established in the literature that higher levels of the flow value of unemployment result in higher levels of finding rate volatility. Since  $b$  is lower now the model must rely more on wage stickiness to generate volatility in the finding rate. The result is that the model slightly underestimates the volatility of the wage. With  $b = 0.65$  the model generates 90% of the observed volatility of wages. When  $b = 0.55$  the model generates 80% of the observed volatility. The wage is also less cyclically responsive. For  $b = 0.65$ , the response of the new hire wage to changes in unemployment is 95% what it was in the benchmark model. The response to productivity is 80% what it was in the benchmark model. For  $b = 0.55$ , the new higher wage is only 85% as cyclical with respect to unemployment and 69% as cyclical with respect to productivity. The volatility of vacancies rises substantially.<sup>19</sup> Finally the separation rate is less correlated with productivity.

### 6.3. Idiosyncratic productivity distribution

In this section I examine the sensitivity of the results to an alternate assumption regarding the idiosyncratic productivity shocks. I use the same value for the idiosyncratic shocks as in the benchmark model but allow for a uniform distribution for the shocks as opposed to a log normal distribution. This modification will make the distribution of shocks more left skewed. Therefore, for the grid  $x_k = \{x_1, \dots, x_K\}$  with  $x_K = x^h$ , I let  $x_1 = 1/K$ ,  $x_k - x_{k-1} = \frac{x^h}{K}$  and set  $\text{Prob}(x_i = x_j) = \tau \frac{1}{K}$  for  $j = 1, \dots, K-1$  and  $\text{Prob}(x_i = x_K) = 1 - \sum_{i=1}^{K-1} \text{Prob}(x_i = x_j)$ .<sup>20</sup> Here  $\tau$  scales down the probability of getting a bad productivity shock. This parameter is necessary because otherwise there would be too high a probability of getting a low productivity shock and the separation rate would be too high. I calibrate the model in the same way as the benchmark model with one exception. The parameter  $\tau$  now replaces  $\sigma^z$  the original standard deviation of the idiosyncratic productivity shocks. The results are contained in Table 8. There is very little difference in the results when using the uniform distribution for idiosyncratic productivity shocks. Importantly, the wage is still as volatile and cyclical as before.

<sup>18</sup> While potentially counterintuitive the intuition is as follows. A higher endogenous separation rate results in fewer jobs at the lower end of the productivity distribution (because they separate). Therefore changes in productivity destroy fewer jobs and the separation rate is linked less to aggregate productivity.

<sup>19</sup> This result is due to the calibration of the model where the elasticity of the finding rate to market tightness is pinned down using the Hosios condition.

<sup>20</sup> I set  $K = 125$  because, unlike with the lognormal distribution, the probability of getting a large negative shock is not declining in the size of the shock. This makes it difficult to match the average separation rate with a large number of negative shocks. This assumption also speeds up the computation.

## 7. Conclusion

Current research evaluating the Mortensen–Pissarides model can be broadly placed into two categories. One set of models uses wage rigidity to create substantial volatility in the job-finding rate, but assumes constant separation rates. Another set of models allows for endogenous separation but omits wage rigidity and does not generate sufficient volatility in the job-finding rate. As a result, neither set of models is equipped to fully explain unemployment fluctuations or more importantly answer the key question as to how much wage rigidity is required to do so. In this paper, I solve a version of the MP model with endogenous separation and wage rigidity. The model is consistent with the volatility of unemployment, the job-finding rate and the job-separation rate while explaining 74% of observed volatility in vacancies. The model also matched the volatility of the average wage and gave reasonable levels of the response of the wage of new hires to productivity and unemployment. I show that a version of the model where the job-separation rate is constant fails to fully explain the volatility of unemployment explaining only 70% of the fluctuations in unemployment.

There were a few key shortcomings of the model. First it was unable to fully match the volatility of vacancies. Second, it could not fully match the strong negative correlation between unemployment and vacancies. Perhaps extending the model to include on-the-job search would improve these shortcomings. Additionally, the model implies correlations with productivity that are substantially higher than in the data. Adding additional shocks to the model may make the model more realistic in this regard.

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