Millimeter-wave achromatic half-wave plate

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We have constructed an achromatic half-wave plate (AHWP) suitable for the millimeter wavelength band. The AHWP was made from a stack of three sapphire a-cut birefringent plates with the optical axes of the middle plate rotated by 50.5 deg with respect to the aligned axes of the other plates. The measured modulation efficiency of the AHWP at 110 GHz was 96 ± 1.5%. In contrast, the modulation efficiency of a single sapphire plate of the same thickness was 43 ± 4%. Both results are in close agreement with theoretical predictions. The modulation efficiency of the AHWP was constant as a function of incidence angles between 0 and 15 deg. We discuss design parameters of an AHWP in the context of astrophysical broadband polarimetry at the millimeter wavelength band. © 2005 Optical Society of America

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1. Introduction

Half-wave plate (HWP) retarders are used extensively for polarimetric measurements. The technique is used across a broad range of electromagnetic frequencies because it provides an effective way to discriminate against systematic errors. The modulation efficiency of a HWP that is constructed from a single birefringent plate can reach 100% for a set of discrete electromagnetic frequencies, but away from these frequencies the efficiency drops rapidly. To overcome this limitation it has been proposed to stack several birefringent plates with specific relative angles of their optical axes.[1–3] Such a construction has been called achromatic HWP (AHWP) because it has a broader frequency range over which the polarimetric efficiency is high compared with a HWP that is made from a single plate. The efficiency of an AHWP depends on the number of plates in the stack and on their relative angles. The concept has been demonstrated experimentally in the optical and IR bands.[4] Murray *et al.*[5] have described briefly measurements of an AHWP made of 5 quartz plates for wavelengths between 350 and 850 μm and an AHWP made of 3 quartz plates for wavelengths between 1 and 2 mm. No detailed information is given about the measurements, the analysis, the tests for systematic errors, or the optimization of the AHWP in respect to the relative angles between the plates. A two-element achromatic waveguide polarizer for operation at ~1 cm is mentioned by Leitch *et al.*[6]

In this paper we present the construction of a sapphire AHWP and measurements of its properties at a wavelength of 2.7 mm (110 GHz). We also present an analysis of the design of a three-plate sapphire AHWP for a wavelength of 2 mm. There is currently interest in an AHWP that is suitable for the millimeter-wave band because of the increase in experimental efforts to measure the polarization of the cosmic microwave background radiation. Several experiments will use HWPs, and increasing the bandwidth where the efficiency is high will increase the signal-to-noise ratio of the experiment.

2. Experimental Setup

A top-view sketch of the experimental setup is shown in Fig. 1. We used a Gunn oscillator[7] at 110 GHz and a diode detector[8] as a source and detector of radiation, respectively. Both source and detector had conical horns that provided beams of 12 deg full width at half-maximum. They emitted and were sensitive to linearly polarized radiation with a −15-dB maximum level of cross polarization at ~10 deg from peak gain. We aligned the source and detector by maximizing the signal received by the detector as a function of its orientation relative to the fixed orientation of the source.

We used wire grid polarizers to increase the level of

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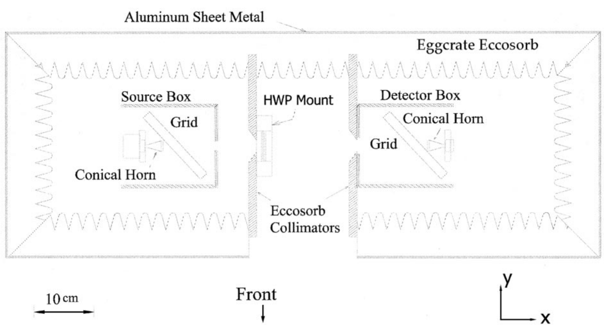

Fig. 1. Top view sketch of the experimental setup.

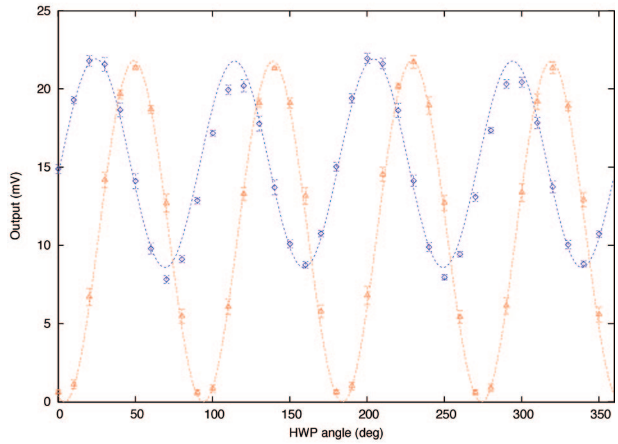

Fig. 2. (Color online) Measurements (points) and theoretical predictions (dashes) of the signal detected as a function of rotation angle of the plates for the chromatic plate (diamonds) and for the AHWP (triangles). Error bars are the standard deviations of five repeated measurements. The theoretical predictions have no free parameters.

linear polarization of the light emitted by the source and detected by the detector. The grids, which were made by Buckbee–Meers, were measured to have a modulation efficiency of 97%.[9]

The source, detector, and polarizers were housed in metallic boxes that were lined inside (outside) with Emerson and Cuming Eccosorb LS-16 (LS-14). One side of the boxes was open.

Between the boxes were placed two 1.25-cm-thick plates of Emerson and Cuming Eccosorb MF-124 that served as collimators. They had 19-mm-diameter knife-edged holes that faced the source. The knife edges were covered with 0.07-mm-thick aluminum tape. The HWP was installed between the collimators in a 5-cm-diameter Newport mount that could rotate around the x axis with a resolution of 1 deg. The mount was held by a cylindrical leg that gave it another degree of freedom for rotation around the z axis. The beam filled the central 15% area of the HWP. Its angular extent when it reached the detector was 2 deg.

The entire experiment was mounted on a metallic optics bench. Aluminum sheet metal lined with eggcrate Eccosorb-CV3 enclosed the experiment from three sides. Eggcrate Eccosorb was also placed both in front and above the source and detector boxes, as shown in Fig. 1.

3. Achromatic Half-Wave Plate

We used a stack of three sapphire a-cut plates to construct the AHWP. Each of the fine-ground plates had a thickness of 2.32 ± 0.05 mm, which made each a HWP for a frequency of 193 GHz. The three plates were mounted together with a front and back antireflection coating made of 0.35-mm-thick polished Herasil. The orientation of the second plate was rotated by 50.5 deg with respect to the orientation of the aligned first and third plates. We had an angular accuracy of ± 1 deg in assembling the stack and an accuracy of ± 1.5 deg in orienting the stack-mount normal to the incoming beam. The ordinary and extraordinary axes of any of the plates were known to within 0.5 deg.

We compared the performance of the AHWP with the performance of a chromatic plate, a single a-cut plate of sapphire with a thickness of 2.32 mm. The chromatic plate was stacked with the same layers of antireflection coating as the AHWP.

We used a frequency of 110 GHz to make the measurements because at this frequency the difference between the modulation efficiency of the AHWP and of the single plate are nearly maximized. Therefore a clear demonstration of the achromaticity of the stack is provided.

4. Measurements, Analysis, and Results

To quantify the efficiency of the plates, we measured the detected intensity as a function of their rotation angle α about the x axis. Data were taken angles of 10 deg and are shown in Fig. 2. Error bars are the standard deviation of five repeat measurements of the efficiency. A repeat measurement consisted of assembling all individual pieces into a stack, mounting the stack, and taking data. No changes in other elements in the experiment were made between repeat measurements.

A constant offset of approximately 0.7 mV was measured when the aperture of the detector box was blocked, and then it was subtracted from the data. This level was constant with rotation of the plates, between different independent measurements of a given stack and between measurements with different stacks. The data was then fit with the following model:

$$D = \sum_{i=0}^{8} A_i \cos(i\alpha + \phi_i). \tag{1}$$

The output of the fitting were the nine amplitudes and eight phases, where ϕ_0 was set to zero. The modulation efficiency was defined as

$$\varepsilon = \frac{A_4}{A_0}. \tag{2}$$

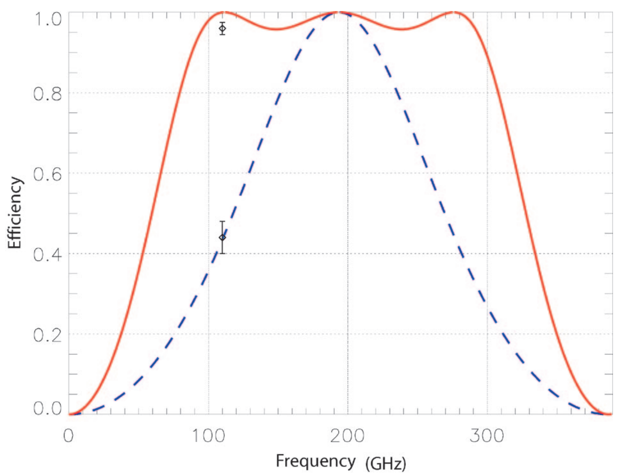

Fig. 3. (Color online) Predicted modulation efficiency as a function of frequency of the AHWP (solid curve) and of a single plate (dashed curve) and the measured efficiency of both plates.

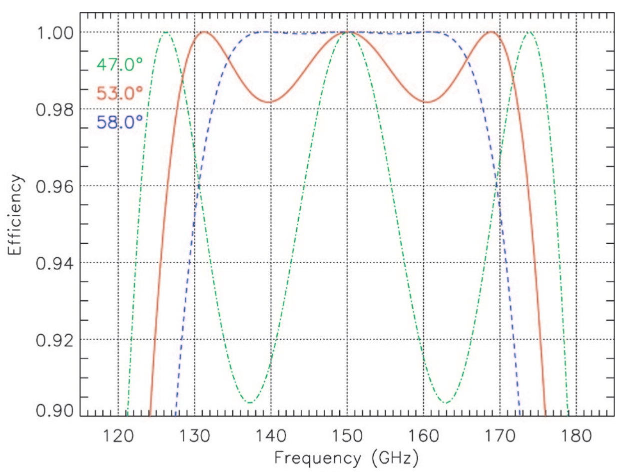

Fig. 4. (Color online) Predicted modulation efficiency of an AHWP as a function of frequency near 150 GHz for rotation angles of 47 (dash-dotted curve), 53 (solid curve) and 58 (dashed curve) deg of the second plate. Each sapphire plate in the stack is a HWP for a frequency of 50 GHz.

The value of ε did not change when we fit the data only up to the fourth harmonic (five amplitudes and four phases). The quality of the fit, however, degraded from a reduced χ^2 of 0.27 and 0.9 for the achromatic and the chromatic plates, respectively, with 8 harmonics to 5.8 and 2.6, respectively, with 4 harmonics.

Predictions about the efficiency of the plates were calculated with the technique of Mueller matrices. The intensity of the light incident on the detector was generated by multiplying an incident Stokes vector representing 100% Q-polarized light by Mueller matrices that simulated the response of the two antireflection layers, the plates, and a 100% Q-polarized detector. An overall normalization was taken from a measurement of the power detected in the absence of a HWP in the light path. The phase was taken from the known orientation of the plates. Normal incidence was assumed throughout. A prediction for the detected intensity was calculated as a function of α in steps of 1 deg, fitted by the model given in Eq. (1), and a predicted efficiency was calculated with Eq. (2). The predicted response of the plates as a function of angle is shown in Fig. 2. The prediction shown is not a fit to the data. There are no free parameters in this prediction.

Figure 3 shows the predicted efficiency of the chromatic and the achromatic plates as a function of frequency and our measured values of 43 ± 4% and 96 ± 1.5%, respectively. The predicted values are 43.5% and 100%, respectively. Uncertainty in the predicted values of the efficiency, due to uncertainty in the indices of sapphire,[10] is 1.5% for the single plate and is negligible for the AHWP. The errors on the measurements of the modulation efficiency were calculated by summing the statistical and an estimate of the systematic errors in quadrature.

We also measured the efficiency for angles of incidence that are not normal by tilting the AHWP about the z axis between angles of 0 and 15 deg. We found no change in the efficiency as a function of angle within statistical errors.

Spurious signals generated by reflections can be a source of systematic errors. We checked the level of signal detected by the detector when either of the collimators were blocked with metal or with a piece of Eccosorb MF124. The level was 0.7 mV for all cases and did not change as a function of the rotation angle of the plates in their mount.

The experiment was repeated for various distances of the plates from the source. The efficiency of the single plate varied in a sinusoidal manner with position with an amplitude of 1.9% and a period of 1.4 mm. This period is also half the wavelength of the source, and we hypothesize that reflections in the setup cause the small variation in efficiency. We have also observed that the shape of the deviations between the theoretical prediction for the detected signal and the one measured vary as a function of the position of the plate. The data shown in Fig. 2 are representative of the magnitude of such deviations. For the achromatic plate the peak-to-peak changes in efficiency as a function of distance were smaller than the quoted statistical error.

5. Discussion

There is good agreement between each of the no-free-parameters predictions shown in Fig. 2 and the data. Both the predicted overall modulation amplitude and the relative phase shift are reproduced by the measurements. The measured modulation efficiencies are close to the predicted values.

AHWPs can be constructed with various combinations of birefringent plates, each giving a different degree of achromaticity. Title[3] showed that with three plates of the same material an AHWP should have the first and the last plates aligned, and most of our discussion is restricted to such a stack. Figure 4 shows the efficiency of an AHWP made of three sap-

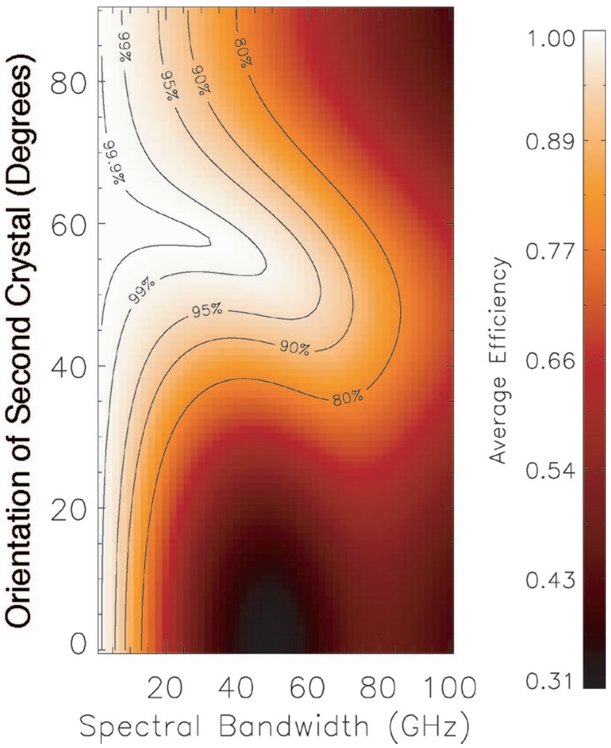

Fig. 5. (Color online) Average modulation efficiency (color scale and contours) as a function of the orientation of the second plate and the spectral width of a top-hat band centered on 150 GHz (for example, a width of 60 GHz means 150 ± 30 GHz).

phire plates as a function of frequency and for three different orientations of the second plate. Each of the plates is a HWP at odd harmonics of 50 GHz, suitable for a cosmic microwave background polarization experiment—E and B EXperiment (EBEX)—that we are currently constructing. EBEX will operate at 150, 250, 350, and 450 GHz. An orientation angle of 58 deg gives close to a constant modulation efficiency over a band of ~40 GHz. A plate orientation angle of 47 deg gives a band of ~60 GHz at the expense of variations of the efficiency within that band. It is therefore interesting to quantify the average modulation efficiency as a function of bandwidth and as a function of rotation angle of the second plate. The results are shown in Fig. 5 for a top-hat frequency response, and they demonstrate several features. The maximum average efficiency decreases as a function of bandwidth, but with a proper choice of angle, average efficiencies that are larger than 95% are achievable with up to 60 GHz of bandwidth. The angular precision required for the orientation of the second plate is rather coarse. The efficiency for 60 GHz of bandwidth is larger than 95% for any angle between 47 and 56 deg. Even smaller accuracy is required for narrower bandwidths.

A stack of five plates can give high modulation efficiency over an even broader range of frequencies compared with a three-stack; see Figs. 6 and 7. With an assumption of a top-hat frequency response of the instrument, we calculate that for the balloonborne EBEX the penalty in increased absorption and emission from the thicker stack of sapphire plates would be smaller than the increase in signal and therefore that a properly designed five stack would increase the signal-to-noise ratio of the experiment.

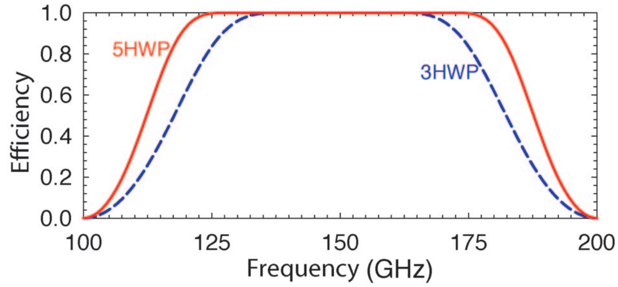

Fig. 6. (Color online) The modulation efficiency of an AHWP made of a stack of five plates compared to the modulation efficiency of an AHWP made of a three stack. The five stack has orientation angles of 28.8, 94.5, 28.8, and 2 deg for the plates after the first. For the three stack the second plate is at 57.5 deg. Each of the plates is sapphire and is optimized for 50 GHz.

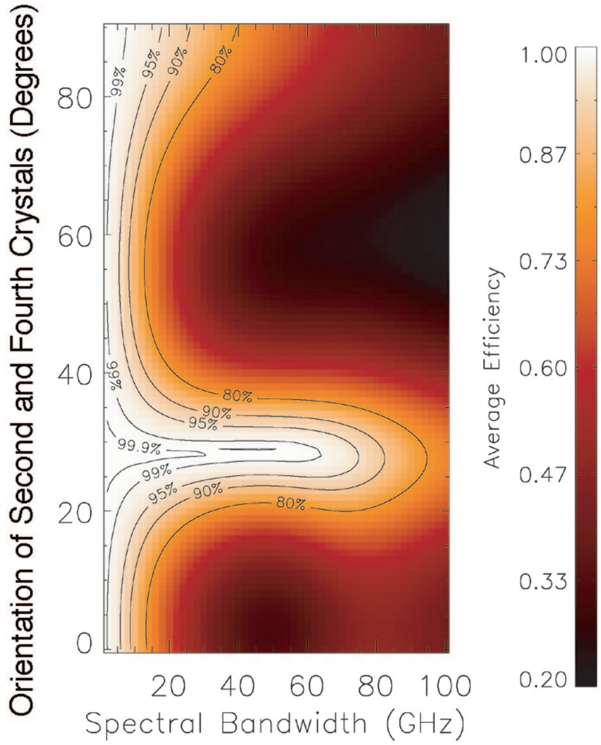

Fig. 7. (Color online) The average modulation efficiency (color scale and contours) for an AHWP made of a five stack. The efficiency is given as a function of the orientation of the second and fourth plates (relative to the first) and the spectral width of a top-hat band centered on 150 GHz. The relative angles of the third and fifth plates are 94.5 and 2 degrees, respectively.

Interest in millimeter-wave AHWP has increased recently because of the scientific interest in the polarization of the cosmic microwave background radiation. In several experiments, including our own EBEX, use of HWPs as a means to modulate the incident polarization is proposed.[11,12] The results presented in this paper provide reassurance that these experiments can rely on an AHWP and that the efficiency of such a plate is constant for a relatively broad range of incidence angles.

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