Homework #1: Answer Key

Economics of Sports 229 Assistant Professor: Victor Matheson

1. a. Set MR = MC. Q = 10,000 - 500P P = 20 - Q/500

   $TR = PQ = Q(20 - Q/500) = 20Q - Q^2 / 500$
   $MR = \partial TR/\partial Q = 20 - 2Q / 500 = 0 \quad ====> \quad 20 = Q / 250 \quad Q = 5{,}000$
   $P = 20 - 5{,}000/500 = 10$
   $Profit = TR - TC = 5{,}000(10) - 0 = 50{,}000$

   b. Since the optimal number is more tickets than UMass is able to sell, they will sell 5,000 tickets at the price the market will bear.

   $P = 20 - 4{,}000/500 = 12$
   $Profit = 5{,}000(12) - 0 = 48{,}000$

   c. Again, set MR = MC. $Q = 20{,}000 - 2{,}000P \quad P = 10 - Q / 2000$
   $TR = PQ = Q(10 - Q / 2000) = 10Q - Q^2 / 2000$
   $MR = \partial TR/\partial Q = 10 - 2Q / 2000 = 0 \quad ====> \quad 10 = Q / 1000 \quad Q = 10{,}000$
   $P = 10 - 10{,}000 / 2{,}000 = 5$
   $Profit = TR - TC = 10{,}000(5) = 50{,}000$

   d. The university must have a way to prevent alumnae, who are willing to pay $12 per ticket, from purchasing student-priced tickets at $5. This can be done by requiring a student ID for purchasing and using student tickets or by making the student seating section undesirable for alumnae in some way (rowdiness, poor sight-lines, etc.). Another way, in general to successfully price discriminate is to sell both individual game tickets for high prices and season tickets for lower prices. This is known as 2nd degree price discrimination.

   e. This one is tricky. An answer that will get you partial credit is this: A profit-maximizing university wants to sell the expensive tickets first. If it maximizes its profits with respect to alumnae, it sells 5,000 tickets at $10. Now the school is left with 9,000 tickets to sell to the students at whatever price the market will bear.

   $P = 10 - 9{,}000/2{,}000 = 5.5$
   $Profit = 10(5{,}000) + 5.5(9{,}000) = 99{,}500$

   The true answer lies in what is called the equi-marginal principle which essentially states that you want to make as much money from the sale of the last student ticket as you would on the sale of the last alumni ticket. You do this by setting the marginal revenue of the two types of tickets equal to one another subject to the constraint that you can only sell 10,000 tickets.

   $\partial TR/\partial Q_s = 10 - 2Q_s / 2{,}000 = \partial TR/\partial Q_a = 20 - 2Q_a / 500$ (1)
   $Q_s + Q_a = 14{,}000$ or $Q_s = 14{,}000 - Q_a$ (2)
   Multiplying (1) through by 1000 gets $10{,}000 - Q_s = 20{,}000 - 4Q_a$

Substituting (2) into (1) gets $10,000 - (14,000 - Q_a) = 20,000 - 4Q_a$ (3)
$5Q_a = 24,000$ $\Longrightarrow$ $Q_a = 4,800$ $P_a = 20 - Q_a/500 = 10.40$
$Q_s = 14,000 - Q_a = 9,200$ $P_s = 10 - Q_s/2,000 = 5.40$

Total profit = 4,800(10.40) + 9,200(5.40) = 49,920 + 49,680 = 99,600 (> 99,500)

f. The profit/total revenue equation is now a bit more difficult.
TR = Q(P+6) because the university gets an extra $6 from each ticket sold.

$Q = 10,000 - 500P$ $P = 20 - Q/500$
$TR = Q(P+6) = Q(20 - Q / 500 + 6) = 26Q - Q^2 / 500$
$MR = \partial TR/\partial Q = 28 - 2Q / 500 = 0$ $\Longrightarrow$ $26 = Q / 250$ $Q = 6,500$
$P = 20 - 6,500 / 500 = 7$
Profit = TR - TC = 6,500(7+6) = 84,500

g. The profit/total revenue equation is now even more difficult.
TR = Q(P+6) because the university gets an extra $6 from each ticket sold.
TR = .5QP + 6Q because the university gets an extra $6 from each ticket sold from concessions but must give a portion of the gate to the Conference.
$Q = 10,000 - 500P$ $P = 20 - Q/500$
$TR = .5QP + 6Q = .5Q(20 - Q / 500) + 6Q = 16Q - Q^2 / 1,000$
$MR = \partial TR/\partial Q = 16 - 2Q / 1,000 = 0$ $\Longrightarrow$ $16 = Q / 500$ $Q = 8,000$
$P = 20 - 8,000 / 500 = 4$
Profit to UMass = .5(4)(8,000) + 6(8,000) = 16,000 + 48,000 = 64,000
Profit to A-10 = .5(4)(8,000) = 16,000

Notice that there is dead-weight loss here (total combined profits are 80,000 vs. 84,500) as the revenue sharing structure skews UMass's incentives towards selling more tickets at a lower price in order to secure the non-shared concession revenues.

h. The answer to this lies in the responsiveness of the fans of the teams in the Conference to winning. The incentive of the Conference is simply to sell the most dollars of tickets conference-wide. If ticket buyers are strongly dissuaded by blowouts and runaway conference champions, then parity will likely increase Conference revenues. On the other hand, if there are many teams who sell as many tickets for a winner as a loser, but a couple of teams only buy tickets when their team is a champion, the Conference is best served by having those teams win all of the time. Similarly, if a several teams have large arenas and large fan bases, while others are very small, it is likely that Conference revenue is maximized by having the big teams win most of the time.

i. They could impose recruiting rules that are binding on all teams, require all teams to have the same number of rostered players, and have all teams have the same number of scholarships. They could also accommodate scheduling so that all teams have the same number of home and away games. They could require that all teams have some minimum standard of athletic facilities. For example, all schools might need a gymnasium large enough to accommodate a certain number of paying fans so that all teams could field the maximum number of players.

j. Because each of the colleges in the conference is controlled by independent boards of trustees and since the conference has no ownership stake in any of the schools, the situation is best described as a franchise.

2. a. The inherent problem is that winning the 8th match in a Sumo tournament (out of 15 matches) results in a disproportionate return compared to winning either 7 or 9 matches. Thus, a wrestler coming into his final match with 7 wins has about 4 times more to gain from a win than a wrestler coming in with any other number of wins has to lose. According to the authors, winning an 8th match increases a wrestler's annual earnings by about $33,000 while the losing wrestler would only suffer about $10,000 in losses.

b. There is lots of evidence here but the easiest to understand is simply that a disproportionate number of wrestlers end up with exactly 8 wins. While a binomial distribution suggests that about 19.6% of wrestlers should end up with either 7 or 8 wins in a 15-match tournament, in fact, 26.0% end up with exactly 8 wins and 12.2% end up with exactly 7 wins while all other exact numbers match up pretty close with the binomial distribution.

c. While statistics can never prove something isn't coincidence, the authors show that the chance of something like this occurring through random chance is extremely low. (The actual numbers are something like 1 in 10xE36.) In addition, several attempts are made to distinguish match fixing from effort. For example, the next time wrestlers meet after a "suspect" win, the previous winner loses a disproportionate amount of time. In addition, the statistics are even more skewed among wrestlers from the same organization or between wrestlers suspected from other sources to be corrupt.

d. There are tons of examples here including teams tanking to get better draft picks, players slacking if they have long-term contracts, teams not playing as hard at the end of the season if they have already qualified for or been eliminated from the playoffs, and any connections between athletes or officials and gambling.