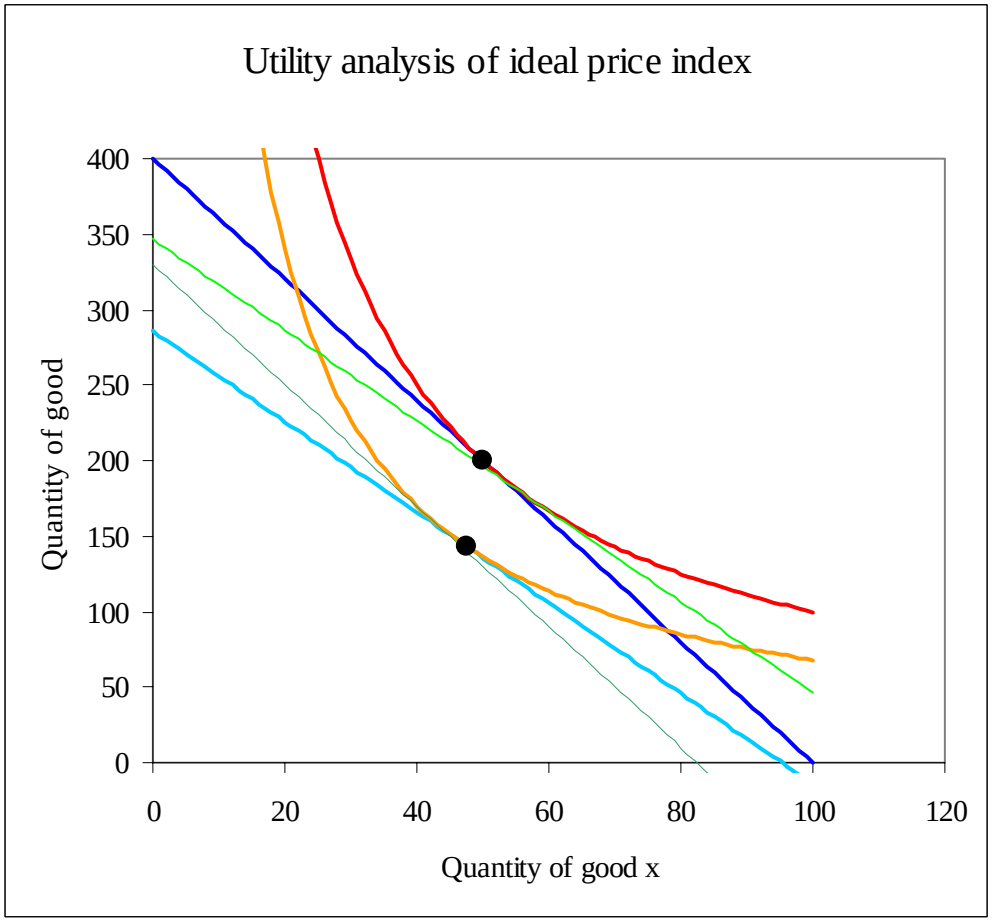
Utility analysis of ideal quantity index

A true ideal quantity index (in percentage terms) can be expressed as the percentage of income taken away from a consumer at initial conditions (prices and income) necessary to leave the consumer at the same level of utility after the price change. Alternatively, it can be expressed as the percentage of income given to a consumer after the price change necessary to leave the consumer at the same level of utility after the price change.

On a two-good indifference curve analysis diagram, the ideal index is represented by the shifting budget constraint after taking into account the relative price changes. Consider the graph below (the spreadsheet used to create the diagram may be downloaded from the supplemental web page). The dark blue line is the budget line for the case when p_x = \$4.00, p_y = \$1.00, and I = \$400. The red indifference curve is tangent to the budget line at the optimum, x = 50, y = 200. Now consider a price change to p_x = \$4.20, p_y = \$1.40, with I = \$400. This price change has decreased the price ratio from p_x/p_y = 4 to 3. The new budget constraint is the light blue line, and the orange indifference curve is tangent to this budget line.



The question is, "how are we to value the lost income?" If we simply take the difference in the tangent points, we are allowing for a substitution effect: the consumer shifts her relative consumption towards good x, whose relative price fell. To account for this, we use the principles of true ideal price index described above.

Consider the first definition: "the … income taken away from a consumer at initial conditions (prices and income) necessary to leave the consumer at the same level of utility after the price change." Here we use the price ratio of the initial condition ($p_x/p_y = 4$) and find the income level that just meets the new (orange) indifference curve (which represents the level of utility after the price change). This is the dark green budget line. Thus, the difference between the dark blue and dark green lines represents the lost income.

Consider now the second definition: "the … income given to a consumer after the price change necessary to leave the consumer at the same level of utility after the price change." Here we use the price ratio of the new conditions ($p_x/p_y = 3$) and find the income level that just meets the old (red) indifference curve (which represents the level of utility before the price change). This is the light green budget line. Thus, the difference between the light green and light blue lines also represents the lost income.

For certain types of utility functions (i.e., when preferences are homothetic), the two definitions will give identical results. However, if preferences are not homothetic, the two definitions will yield different results. In these cases there is no single ideal quantity (or price) index.

Acknowledgement

The author is grateful to an anonymous referee for the spreadsheet example used to motivate this discussion.