Biology 390 **The Scaling of Metabolism – Solutions to Problems**

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The equation that describes the relationship between rates of oxygen consumption and body size in mammals is:

$\dot{V}_{O_2}$ = 0.676 $M^{0.75}$ (Eq 1)

where: $\dot{V}_{O_2}$ is **liters O_2 / hour** and **M** is body mass in **kilograms**.

1. Derive an equation (from eq. 1) for mammals that expresses the relationship between the **mass-specific** $\dot{V}_{O_2}$ (*i.e.*, liters O_2 hr^{-1} kg^{-1}) and body mass.

Simply divide through by $Mass^{1.0}$

($\dot{V}_{O_2}$ = 0.676 $M^{0.75}$) / M

$\dot{V}_{O_2}$ / M = 0.676 $M^{-0.25}$

Here's a quick check. If we solve both equations (whole animal and mass-specific for some sized animal and if we then divide the whole animal metabolism by the mass, we should get we should get the same answer as from the mass-specific equation. Assume mass = 10 kg:

whole animal: 0.676 * 10^0.75 = 3.8 L O_2 / h; 0.38 L O_2 / (kg h)

mass specific: 0.676*10^-0.25 = 0.38 L O_2 / (kg h)

The equation is correct

2. Derive an equation from eq. 1 where the dimensions for $\dot{V}_{O_2}$ are ml O_2 per hour and **M** is in **grams**. (This is "tricky" so please check your answer by numerical substitution. – you should check the appendix to the scaling notes and Schmidt Nielson before trying this one)

$\dot{V}_{O_2}$ = 0.676 $M^{0.75}$ (Eq 1)

This is a classic case of adjusting the value of the constant.to reflect the new units. Note that the initial constant (eq. above) must have the following units:

0.676 L O_2/ (h * $kg^{0.75}$)

-- the $kg^{0.75}$ cancels with the mass term. As usual, L O_2 has an assumed exponent of 1.

Thus:
(a) Conversion from liters to ml is trivial -- there are 1000 ml in a liter and so we multiply the constant by 1000.
(b) to convert from kg to g we cannot simply divide by 1000 -- instead we must include the exponent and divide by $1000^{0.75}$.

Here goes:

$0.676 \text{ L } O_2 / (h * kg^{0.75}) * 1000 \text{ mL} / 1 \text{ L} * 1 \text{ kg}^{0.75} / 1000 \text{ g}^{0.75} = 3.8 \text{ mL } O_2 / (g * h)$

so:

$\dot{V}_{O_2}$ (mL O_2 / h) = 3.8 $M^{0.75}$ (g)

Let's be careful and see if we get the same answer from the old equation and the transformed one. Once again, let's assume we have a 10 kg animal:

old equation (answer given above at end of last problem) = **3.8 L O_2 / (h) or**
= 3,800 mL O_2 / h

from the "new" equation (using 10,000 g instead of 10 kg):

$\dot{V}_{O_2}$ (mL O_2 / h) = 3.8 $M^{0.75}$ (kg) = 3.8 * $10{,}000^{0.75}$ = 3.8 * 1000 = **3,800 ml O_2 / h**

The transformed equation is correct.

3. Given: the scaling relationship between body mass and $\dot{V}_{O_2}$ in mammals:

$\dot{V}_{O_2} = 0.676 \, M^{0.75}$

where: $\dot{V}_{O_2}$ is **liters O_2 / hour** and **M** is body mass in **kilograms**.
An investigator wanted to measure the effect of a particular habitat on the energetics of a certain group of mammals. He selected two species belonging to the same genus for his comparisons. Species A weighed 10 grams and lived in the habitat in question. Species B weighed 669 grams and lived in a more "typical" habitat. Their measured rates of metabolism were:

Species	Standard Rate of Metabolism ($\frac{\text{liters } O_2}{\text{hr}}$)
A (10 g)	0.0107
B (669 g)	0.250

a. Which species has the higher total rate of metabolism? How many times higher? (*no tricks here, its an easy calculation that is useful for comparison below*)

In terms of whole animal (total) metabolism B has a higher rate, 0.25/.0107 = 23 X greater.

It is also worth noting that species B's mass is 67 X greater

b. Which species has the higher mass-specific rate of metabolism? How many times higher? (*again, no tricks here)*

Species A = 0.0107/10 = 0.0011
Species B = 0.25/669 = 0.0004

in this case, species A is 0.0011 / 0.0004 = 2.9X greater

c. The investigator wants to determine whether living in the particular habit affects rates of metabolism. So , he desires to partition out the effect of body size in his comparisons.

(i) Is it biologically valid to accomplish this by comparing the rates of metabolism divided by weight, *i.e.*, comparing mass-specific rates?

No, they are too different in size. It is know that metabolism and mass do not scale isometricially. A comparison must take into account this fact.

(ii) In what specific circumstances can you use mass-specific rates in comparisons?

When animals are of very similar size -- generally less than and order of magnitude.

(iii) What is the apparent effect of living in this habitat on the rates of metabolism? (Is living in the habitat correlated with an increase or decrease in rates of oxygen consumption?) How much of an increase or decrease? Express your answer in percent.

Here we use the McNab type analysis. First we need to find the predicted rates of metabolism for each animal. The equation is on the last set of problems and in the book (don't worry -- you'd be given the equation on an exam):

$\dot{V}_{O_2}$ = 0.676 $M^{0.75}$ where mass is in kg and $\dot{V}_{O_2}$ is in L O_2 / h

Species A has a mass of 10 g = 0.010 kg; its predicted rate of metabolism is:

0.676*0.01^0.75 = 0.021 L O_2 / h

Its measured rate is 0.0107 L O_2 / h

Thus, its measured rate / predicted rate = 0.0107 / 0.021 = 0.51 -- its rate is 51% of what is expected for a typical animal of its mass.

For species B, mass = 669 g = 0.669 kg

and so its predicted metabolism = 0.676 * $0.669^{0.75}$ = 0.50 L O_2 per hour

Its measured metabolism was 0.25 L O_2 / h

measured / predicted $\dot{V}_{O_2}$ = 0.25/0.50 = 0.5 -- 50% of expected.

These animals both have rates that are half of expected -- thus they are actually metabolically very similar (but very different from the "standard or benchmark" mammal).