# Excitable Cell Physiology -- Problem Set #1:
## Does the Migration of Ions from one Compartment to Another During the Establishment of a Donnan Equilibrium Significantly Affect Ionic Concentrations?

**Review** the concept of capacitance before attempting this problem.

***Assume*** that we have a very simple cell where the cation ($C^+$) is the only diffusible species.
**Further** assume that the concentrations of $C^+$ are:

$[C^+]_{inside} = 0.1$ M
$[C^+]_{outside} = 0.0019$ M

and the temperature is 20° C.

1. Calculate the transmembrane potential for this system at equilibrium. (To make later calculations simple, round your answer to a whole number.)

**Use the Nernst eq: $E_{C+} = -58\ \text{mV} * \log(0.1\ \text{M}/0.0019\text{M})$**

**$E_{C+} = -100$mV (with a bit o'rounding)**

2. The capacitance of a cell membrane is always normalized to area: it is expressed as farads/area. This is obviously necessary since bigger cells with bigger membranes would have larger capacitances (since capacitance is determined by the area of the conductors and the dielectric constant). **For a typical biological membrane, the capacitance is about:**

**1 mF/cm$^2$, i.e., $10^{-6}$ Farads/cm$^2$.**

Recall that 1 Farad = $\frac{\text{coulomb}}{\text{volt}}$

(a) Calculate the amount of charge stored across one cm$^2$ of membrane. Give your answer in coulombs per cm$^2$.

**$Q = C*E = 1 * 10^{-6}$ coulombs/(V*cm$^2$) * 0.1 V**

**$= 1 * 10^{-7} = 0.0000001$ coulombs/cm$^2$**

(b) Calculate the number of mols of $C^+$ per $cm^2$ of membrane. You will need the following constant:
1 Faraday of charge = 1 mol of univalent charged particles = 96,500 coulombs

**# mols $C^+$/ $cm^2$**

**= $(1*10^{-7}$ coulombs/$cm^2$) / ($9.65*10^5$ couls/mol chrg)**

**= $1.04*10^{-12}$ mols of charge /$cm^2$**

**= $1.04*10^{-12}$ mols of charge /$cm^2$ * $6.02*10^{23}$ particles/mol**

**= $6.24*10^{11}$ charged particles /cm**

***This may seem like a lot of particles to have moved across the membrane (given what I told you in class) but lets put it in context:***

(c) Now, assume that for this typical axon, we are concerned with a:
**1 cm($1* 10^{-2}$ m) long section** with a
**diameter of 10 $\mu$ ($10 * 10^{-5}$ meters)**.

Therefore, in this section, the axon has the following **surface area**:

= length * area = L * diameter pi = $1*10^{-2}$ m * pi * $1*10^{-5}$ m

= $3.142*10^{-7}$ $m^2$ = $3.142 * 10^{-3}$ $cm^2$

What is the number of charged particles stored in a membrane capacitor of this surface area?

**Here we use the value from (b) which gives us the number of separated charged particles per $cm^2$ at -100 mV and multiply this by the axon's actual surface area. Thus:**

**= actual area ($cm^2$) * charges per $cm^2$ @ -100 mV**

**= $3.142 * 10^{-3}$ $cm^2$ * $6.24*10^{11}$ charged particles /cm**

**= $1.96 * 10^9$ charged particles**

(d) How many charged particles are present in the cytosol of this same bit of axon (1 cm with a 10 micron diameter)?
The internal concentration of $C^+$ was given at the start as 0.1 M. We can calculate the volume as:

= cross sectional area * length = $(\text{diameter}/2)^2$ * pi * L

= $(5*10^{-5}\ m)^2 * pi * 1*10^{-2}\ m = 7.85 * 10^{-13}\ m^3$

and since 1 cc = 1 ml = $10^{-6}\ m^3$ then the volume is, in ml:

= $7.85 * 10^{-7}$ ml

So, how many $C^+$ particles reside in this tiny volume if the concentration of $C^+$ is 0.1 M?

Let's get everything into ml -- since there are 1000 mL in a liter, then 1 mL contains 1/1000 of the $C^+$ present in a liter -- *i.e.*, in this case there are $1 * 10^{-4}$ mols $C^+$ per ml.

**What is the total number of particles present in this 1 cm by 10 micron axon?**

**mols $C^+$ per ml * actual # of ml * # particles per mol:**

**$1 * 10^{-4}$ mols/ml * $7.85 * 10^{-7}$ ml * $6.02 * 10^{23}$ ions/mol**

**= $4.73 * 10^{13}$ $C^+$ in the section**

(e) What is the ratio of the number of charges separated in the establishment of the Gibbs-Donnan Equilibrium to the # number of ions still inside the cell?

Using the answers from (d) and (e):

**$1.96 * 10^9$ C+ separated / $4.73 * 10^{13}$ $C^+$ inside *100**

**= 0.004%**

(f) In the establishment of a Gibbs Donnan equilibrium, is there any meaningful change in the concentrations of the ions inside the cell?

**Obviously not! See (e)**